SEP 25 192

ELEVENTH ANNUAL MEETING OF THE OPTICAL SOCIETY OF AMERICA
PHILADELPHIA, PA., OCTOBER 21, 22 and 23, 1926.

Volume 13 Number 3

Journal of the Optical Society of America and Review of Scientific Instruments

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Published Monthly by the
OPTICAL SOCIETY OF AMERICA

OFFICE OF PUBLICATION
GEORGE BANTA PUBLISHING COMPANY
450 454 Ahnaip St.,
MENASHA, WISCONSIN

Entered as second-class matter May 31, 1921, at the postoffice at Menasha Wis., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in the Act of February 28, 1925, embodied in paragraph 4, section 412, P. L. and R. authorized June 19, 1925.

Journal
of the
Optical Society of America
and
Review of Scientific Instruments

Published monthly by the Optical Society of America in cooperation with the Association of Scientific Apparatus Makers of the United States of America. The Section on Optics is devoted to original papers on experimental, theoretical, and applied optics. The Section on Scientific Instruments is devoted to original papers on instruments of all kinds (electrical, mechanical, thermal, etc., as well as optical) for research instruction, and industrial processes. Manuscripts for publication are invited, whether authors are members of the Optical Society or not.

Annual subscription $5.00 dating from January of current year. Single copies $0.60. Back numbers: Vols. 1–5 (The Journal of the Optical Society, 1917–1921 inclusive) $16.00. Vol. 6–7, $5.00 per volume; Vol. 8—, $2.50 per volume. "Address publishers, George Banta Publishing Company, 450–454 Ahnaip Street, Menasha, Wisconsin, or F. K. Richtmyer, Manager, Cornell University, Ithaca, New York, for all correspondence regarding subscriptions, back numbers, changes of address, advertising rates and accounts, and missing numbers."

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Vol. 13 SEPTEMBER, 1926 Number 3

THE OPTICAL THEORY OF SKIASCOPY

By James P. C. Southall

Introductory. Skiascopy (or retinoscopy, as it is also called) is a very practical and convenient optometrical method of determining the state of refraction of the eye. As practised by a careful and skillful observer who thoroughly understands the optical theory of the process and appreciates both its limitations and advantages, the method of skiascopy is capable of giving results that are extremely accurate and reliable. Only the simplest and most inexpensive equipment is needed in the way of apparatus, and the findings can be obtained quickly without annoying or fatiguing the patient. Being simply a parallax measurement depending on the relative apparent movement of the reflex in the patient's eye, the state of refraction of the observer's eye is not involved at all, as it is, for example, in the ophthalmoscopic determination of refraction by means of the erect or inverted image of the fundus in the eye; and if (as in so-called "static skiametry" as distinguished from "dynamic skiametry") it is sometimes necessary that the patient's accommodation shall be entirely relaxed, the observation can be made at a distance far enough away from the patient to render it much more likely that this essential condition is being fulfilled. However, the chief advantage in skiascopy, and the reason why it has become almost indispensable in practice, is the fact that it is a purely objective method of refraction and as nearly as possible independent of any information communicated by the patient himself. On the other hand, it must also be admitted that there are some practical difficulties that have not been altogether overcome and certain curious discrepancies that would seem to imply that the complete theory of the method either has not been fully ascertained or at least is not generally

understood. The conditions for distinct vision are undoubtedly more complicated than is ordinarily supposed, and it is very likely that in the so-called "region of neutrality" (which is the most puzzling difficulty that is encountered in the actual practice of skiascopy) various obscure factors of one kind and another are involved which have not been precisely determined; and while ingenious explanations have been offered to account for this phenomenon, advocated sometimes with much plausibility, none of them can be said to be quite satisfactory in view of all the facts.

Obviously, the essential first step that is preliminary to any complete theory of the method of skiascopy is a thorough investigation of the optical principles on which the method is based; and the main purpose that the writer has in view is to formulate the optical problem as simply as possible and to call attention to some of these factors that have not generally received sufficient consideration.

Suppose that a certain area on the fundus of the eye has been illuminated in some fashion by artificial light; then without perhaps being able to see the fundus itself, an observer at some distance away from the patient will be able to perceive this illumination provided some of the light coming from the fundus passes out of the patient's eye and ultimately enters the observing eye. This illumination will have the appearance in general of a luminous disc in the pupillary plane of the eye under examination, which may, however, be partially eclipsed behind the iris. If the source of light, supposed to be at some considerable distance away from the eye, undergoes a slight displacement at right angles to the line of sight, the observer can watch the spot of light as it moves across the patient's face, and when the light falls upon the pupil of his eye, he may be in a position to see the luminous reflex from the fundus of the eye or the so-called "shadow" with the reddish glow that is characteristic of ordinary light after having been reflected from the posterior wall of the interior of the human eye. The reflex begins to appear at one edge of the pupil and "waxes" until it becomes "full" and then "wanes" until it gradually disappears entirely at the opposite edge. In the method of skiascopy the observation consists in watching this "movement of the shadow" and in noting whether it takes place in the same direction as the movement of the spot of light across the face or in the opposite direction; for in general the shadow will move one way or the other, that is, "with" or "against" the movement of the spot, depending partly on various incidental circumstances connected with the mode of illumination, but also depending essentially

on the state of refraction or focusing of the observed eye. Thus according as the movement of the shadow takes place in one direction or in the opposite direction it is theoretically possible to tell immediately whether the focus plane (that is, the plane, let us say, that is conjugate to the retina of the eye) is or is not between the patient and the observer and thereby to obtain some clue as to the character of the particular optical system that is being investigated at that time. The movement of the reflex is more rapid under some circumstances than under others, but with sufficient practice the observer can acquire great skill in detecting it and in modifying the conditions so as to vary the movement in any desired way and especially so as to cause the direction to be reversed. Incidentally, it may be remarked that neither "skiascopy" nor "skiametry" seems to be a very appropriate name, because it is not so much the "shadow" itself as the luminous reflex that is actually observed; and the term "retinoscopy" is even more unsatisfactory, inasmuch as there is no observation here of the retina or fundus of the eye. "Fundus-reflex test" is rather awkward, but at least it conveys a more accurate idea.

Ordinarily, the eye is illuminated by light reflected from a ***mirror***, which may be plane or curved; and instead of moving the source of light itself, its image in the mirror is displaced by slightly tilting the mirror. Other things being the same, the question as to whether the movement of the reflex is "with" or "against" that of the spot of light on the face will depend simply on whether the image of the source of light in the mirror is or is not formed in the space between the patient and the observer. The mirror may be simply a small piece of thin transparent glass that not only enables the observer to look through it and watch the luminous phenomenon in the patient's eye, but also enables the patient to see the observer's eye; or it may be an opaque silvered mirror, in which case it should contain a tiny round opening not more than about 2 mm in diameter made by removing the silvering at the center of the mirror, to serve as a ***peep-hole*** for the observer, whose eye is supposed to be just behind the mirror and as close to the peep-hole as possible. As a matter of fact, the mirror is usually a ***plane mirror***, and hereafter it will be tacitly assumed that this is the case. The little instrument called a ***skiascope*** (frequently referred to simply as "the mirror") consists of a circular mirror generally about 2 cm in diameter mounted on a metal disc of double this diameter, the surface of the mirror being flush with that of the disc, and hole, mirror and disc being all concentric. The disc is provided with a convenient handle by which

the observer can hold it close in front of his eye supported partly against his brow and the side of his nose, so that no extraneous light can disturb his vision while he is gazing through the peep-hole and watching the reflex in the patient's eye and the spot of light on his face. With this form of mirror skiascope, the *lamp* or source of light must be situated in front of the mirror, and it should be about on a level with the patient's eye and not far from his pupillary plane on the temporal side. In ordinary skiascopy the lamp itself is not very important, and some practitioners continue to prefer an Argand burner with an asbestos screen fitting over the cylindrical glass chimney and containing an iris diaphragm or small round hole directly opposite the light itself, which is usually about 1 cm in diameter, although it can be adjusted to half that size. The virtual source of illumination is the image of this luminous opening in the mirror, and by tilting the mirror slightly vertically or horizontally or in any oblique meridian, the source of light will be caused to execute a corresponding movement. The best way to produce this motion is to hold the skiascope firmly pressed against the eye and tilt the entire head in the given meridian. The image of the source of light in the plane mirror and the source itself will always lie on the circumference of a circle in a plane at right angles to the axis of rotation, the center of the circle being at the center of the hole in the mirror on the supposition that the axis of rotation passes through this point. As the source of light is comparatively close to the optical axis of the observed eye, and as the mirror is supposed to be tilted through a very small angle, the circular arc traversed by the image in the mirror may be regarded as a short segment of a straight line perpendicular to the optical axis. This is an important detail in the geometrical theory of skiascopy, because it implies that all the angular magnitudes are relatively so small that the same graphical methods can be employed for constructing the diagrams as are used, for example, in deriving the formulas for optical imagery in the case of so-called paraxial rays. In other words, the scale of the drawing at right angles to the axis of the optical system is to be considered as enormous as compared with the scale of the linear dimensions parallel to the axis.

The optical theory of skiascopy is much simpler than that of ophthalmoscopy for various reasons, but mainly because in the latter the paramount consideration is the condition for obtaining a distinct optical image of the fundus of the eye that can be focused on the retina of the observer's eye, whereas in the case of skiascopy all that is required is that the observer shall perceive the movement of a more or less vague

reflex from the fundus of the observed eye. Without seriously affecting the generality of the discussion, it may be assumed, at first at any rate, that all the light involved both in illuminating the eye and observing its effects proceeds so close to the optical axis that there is always perfect point-to-point correspondence between object and image, according to the laws of paraxial imagery. If the source of light itself were simply a luminous point, the actual rays entering the eye would all be comprised within a narrow cone whose base would be the entrance-pupil and whose vertex would be at the point S which is the image of the point-source in the mirror. These rays, after being refracted first by the cornea and then by the crystalline lens, will emerge finally into the vitreous humor as a cone of rays whose base is the exit-pupil and whose vertex is at the point S′ which is the optical image of the point S as formed by the dioptric system of the eye. Strictly speaking, the entrance-pupil and exit-pupil are on opposite sides of the actual pupil, but they are so nearly coincident in both position and size that they may be regarded here as identical with the pupil itself. Now unless the transversal plane perpendicular to the optical axis which contains the virtual point-source S happens to be conjugate to the retina of the eye, the focus S′ of the cone of emergent rays in the vitreous humor will not lie on the retina; and in that case the illuminated area of the fundus will be the approximately circular section in which this cone is cut by the retina; and as the mirror is tilted slightly to and fro in some given meridian, this illuminated region will move across the fundus of the eye in a corresponding manner.

Having thus considered the *illumination system* in this elementary fashion, we may proceed now to the *observation system* which includes the illuminated area of the fundus, the optical system of the patient's eye and the peep-hole in the mirror where the observer's eye is placed; for we shall also assume, in this first approximation at least, that any light that passes through this hole will enter the observer's eye, so that we shall not have to take the latter into account. In fact, there is no reason why we should not consider at first that the observer's eye is concentrated, so to speak, into a single point at the center of the peep-hole; although, of course, a fiction of this kind is something that must be kept steadily in mind. While, therefore, the size of the hole in the mirror should certainly be less than that of the pupil of the observer's eye, on the other hand, it should not be so small that the observer will be unable to watch the movement of the spot of light on the patient's face. However, so far as the reflex phenomenon itself is concerned, the

field of view of the observation system may be considered as the region included within the cone whose cross-section in the pupillary plane is the pupil of the patient's eye, and whose vertex is at the center C (Fig. 1) of the peep-hole. Light issuing from the illuminated area of the fundus and traversing the vitreous humor is refracted first through the crystalline lens and then by the cornea so that it emerges finally into the air; and those emergent rays whose paths are contained wholly within the conical region of the field of view of the observation system will go through the hole in the mirror and enter the observer's eye; and thus he will perceive the reflex apparently in the pupillary plane of the patient's eye.

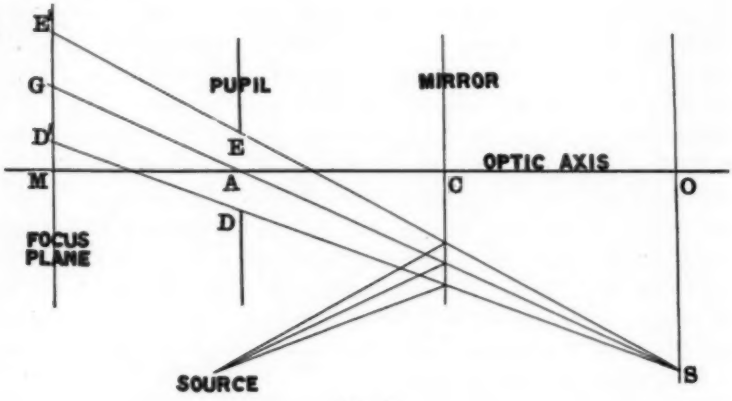

FIG. 1.

The *focus plane* of the observed eye is the transversal plane perpendicular to the optical axis at the *focus point* M which is conjugate to the surface of the retina; and an optical image of the illuminated area of the fundus will be formed, therefore, in this plane, which will be real and inverted or virtual and erect depending on the position of the focus point. Thus if the distance of the center A of the pupil from the focus point is denoted by u, that is, if $u = \mathrm{MA}$, then (on the supposition that the *positive direction along the axis* is the direction in which the patient is looking) a positive value of $u (u > 0)$ will mean that the image in the focus plane is virtual, whereas a negative value of $u (u < 0)$ will generally mean that the image is real. The focusing or state of refraction of the eye may be measured by the reciprocal of the abscissa u, that is, by the

magnitude $U = 1/u$, where the value of U will be expressed in dioptries provided u is measured in meters.[1]

While the whole purpose of the method of skiascopy is to determine the value of U and therefore to find the position of the focus plane, it is perhaps important to emphasize again the fact that the observer does not actually ever see the optical image of the fundus. On the contrary, his whole attention is concentrated on the apparent movement of the reflex in the pupillary plane, which enables him to infer where the focus plane must be and thence to determine the state of refraction of the optical system.

When the patient's accommodation is relaxed entirely, the focus plane and focus point M are the *far point plane* and *far point* of the eye, respectively; and according as the far point does or does not coincide with the infinitely distant point of the optical axis, the eye is said to be *emmetropic* or *ametropic*, respectively. In the latter case when the far point is at a finite distance, the eye will be *myopic* or *hypermetropic* according as this point lies in front of the eye or behind it.

For any definite position of the focus plane the optical image of the illuminated area of the fundus corresponding to a given location of the luminous point-source S may easily be constructed geometrically. Through the center A and the focus point M draw a pair of straight lines in the meridian plane of the diagram (Fig. 1) perpendicular to the optical axis to represent the pupillary plane and the focus plane, respectively. On the first line take two points D and E at equal distances from A on opposite sides of the axis, and let $DE = 2DA = 2AE$ represent the diameter of the pupil in the meridian plane. If the points where the straight lines SD, SA and SE cross the focus plane are designated by D′, G and E′, respectively, the optical image of the illuminated area of the fundus will be the circle described in this plane around G as center with radius $GE' = D'G$.

The central projection of this circle on the pupillary plane from the point C where the observer's eye is situated is found by locating the points D_1, K and E_1 (Figs. 2 and 3) where the straight lines CD′, CG

[1] When the refraction of the eye is measured from the center of the entrance-pupil, the image equations are:

$$m^2U' = U + mF, \qquad my'U' = yU,$$

where U' denotes the reciprocal of the "reduced" distance of the retina from the center of the exit-pupil, F denotes the refracting power of the optical system, and m and y'/y are the values of the magnification ratios with respect to the center of the entrance-pupil and the focus point, respectively. In the case of the unaided eye the value of m is very nearly constant for all focusings of the eye and equal to about +0.9. When the eye is reinforced by a spectacle glass, the magnitudes U and U' must be measured from the pupils of the compound optical system, and F denotes the refracting power of the compound system.

and CE′ cross the pupillary plane, and describing a circle in this plane around the point K as center with radius $D_1K = KE_1$.

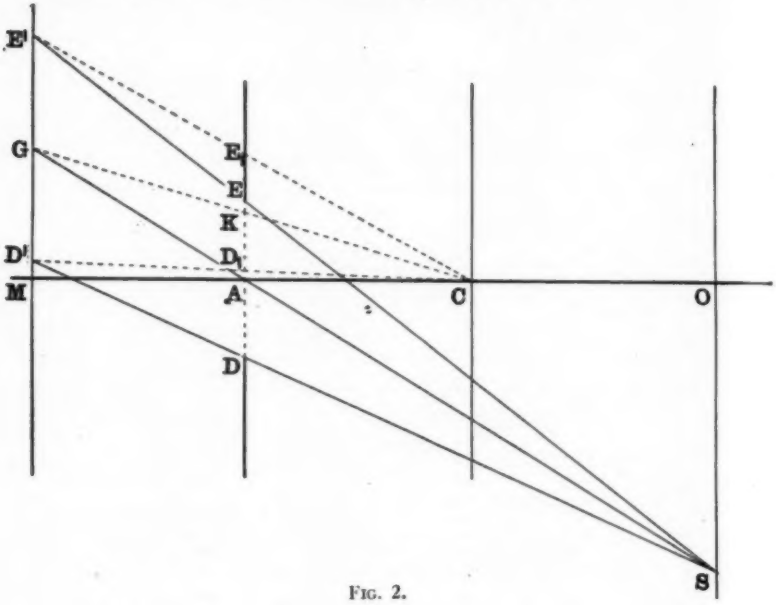

FIG. 2.

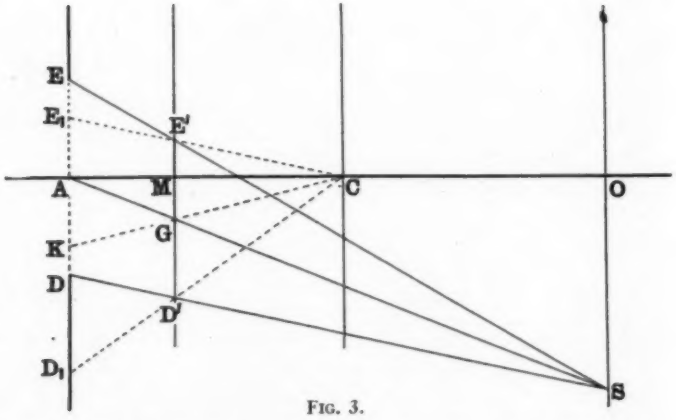

FIG. 3.

Thus far it has been assumed that the source of the light was a luminous point whose image in the mirror was the point that has been designated by S; but as a matter of fact in ordinary skiascopy the source

of the illumination, so far from being a point-source, really has very considerable dimensions, being usually as much as one centimeter in diameter, and therefore as a rule at least several times larger than the diameter of the pupil of the patient's eye. Accordingly, let us suppose now that the source of light is a luminous circle whose image in the plane mirror will also be a circle of the same size with its center at the point S (Fig. 4); and let the ends of the diameter of this image in the

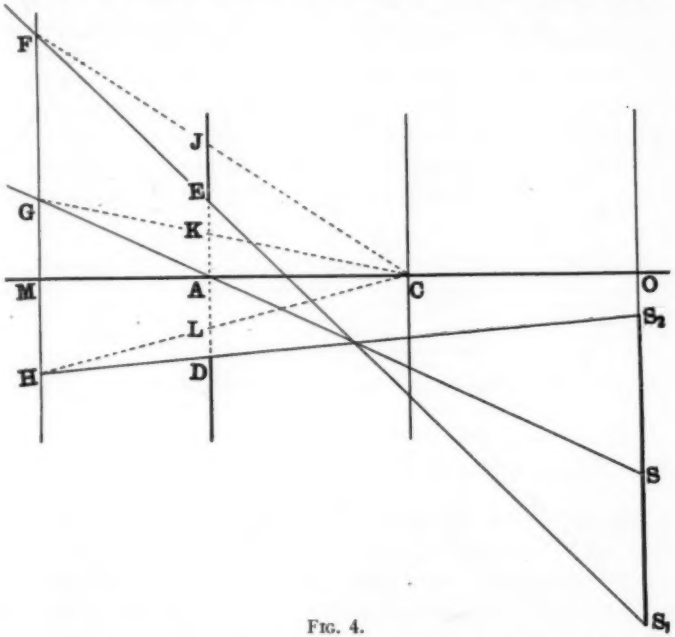

FIG. 4.

mirror in the meridian plane be designated by S_1, S_2, the length of the radius being therefore $S_1S = SS_2$. The region of radiation of the illumination system in this case will be determined by the straight lines S_1E and S_2D drawn from S_1 and S_2 to the opposite ends of the diameter DE of the pupil of the patient's eye and will be comprised within the conical region obtained by revolving these lines around the straight line SA as axis. If the points where the straight lines S_1E, SA and S_2D intersect the focus plane are designated by F, G and H, respectively, the projections of these points on the pupillary plane from the point C as center of projection will be the points J, K and L where the straight lines CF, CG and CH, respectively, cross the pupillary plane; and the

so-called "reflex circle" will be the circle described in this plane around the point K as center with radius $r = LK = KJ$.

Now if any portion of this luminous "reflex circle" happens to be within the otherwise dark area of the "pupil circle" described around A as center with radius $p = DA = AE$, the reflex phenomenon will be visible to the observer with his eye at the center C of the peep-hole in the mirror; and as the mirror is tilted slightly around an axis passing through C perpendicular to the meridian plane, thereby causing the spot of light reflected from the mirror on the patient's face to move

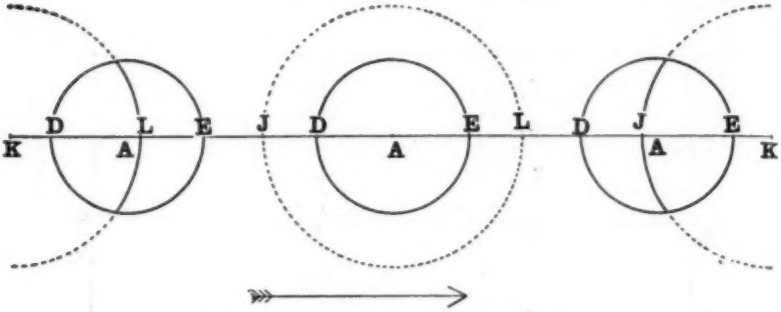

FIG. 5. *Emmetropia, observed at a distance of one meter.*

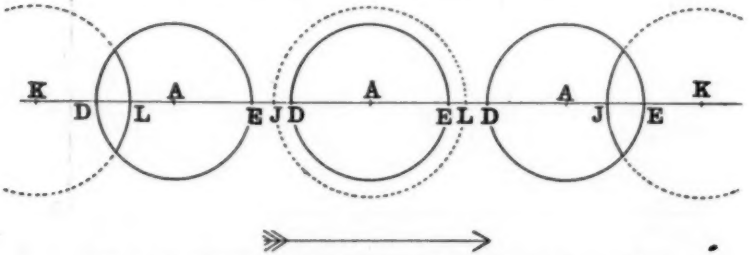

FIG. 6. *Hypermetropia, + 2 dptr, observed at a distance of one meter.*

towards the edge of his eye and then across the pupil itself, the reflex will begin to appear at the instant when the "reflex circle" is tangent externally to the "pupil circle," and will "wax" as the two circles overlap more and more, becoming "full" when they are tangent to each other internally; subsequently the reflex will begin to "wane" until at last it vanishes entirely at the opposite edge of the pupil. The size of the "reflex circle" will depend on various factors and, other conditions remaining the same, it will depend especially on the position of the focus plane, being either larger or smaller than the "pupil circle"

as illustrated in the accompanying diagrams (Figs. 5, 6 and 7), which have been drawn for various special cases of an eye with accommodation relaxed and with no lens interposed in front of it.

The center of the spot of light on the patient's face will be at the point B where the straight line SC drawn from S through the center of the mirror meets the pupillary plane (assuming, for the sake of simplicity, that the surface of the face is in this plane); and as the mirror is tilted slightly so as to cause the point S to move towards the point O where the straight line drawn through S perpendicular to the optical axis crosses this axis, the point B will execute a corresponding movement towards the center A of the pupil; and provided the focus plane M does not lie between A and C, the center K of the "reflex circle" will move in the same direction as the spot of light or "with the mirror"; whereas if the focus plane happens to be between A and C, the movement of the reflex will be "against" the mirror.

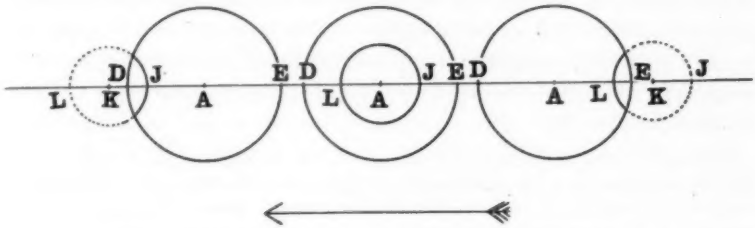

FIG. 7. *Myopia, −2.5 dptr, observed at a distance of one meter.*

The nearer the focus plane is to the mirror, that is, the shorter the interval between the focus point M and the center C of the peep-hole, the larger will be the size of the "reflex circle," and in the limiting case when the points M and C are actually coincident, the diameter of this circle will be infinite. This particular condition when the patient's eye is focused on the mirror itself can be determined, theoretically at least, with very great precision, because then the reflex should appear to be absolutely stationary (although in reality its speed will be infinite), as the mirror is tilted one way or the other in the given meridian. This special position of the focus point M is called the *neutral point* (or *point of reversal* of the movement of the reflex); and as skiascopy is usually practised, the method consists simply in so adjusting the optical system that there is no movement of the "shadow." Thus, for example, when the patient's eye was fixed on a point M somewhere in front of it, it might be possible for the observer, without disturbing the state of

refraction of the patient's eye, to vary his own position with respect to the patient, until he succeeded in locating the place where there was no movement of the "shadow." But apart from other practical difficulties about this mode of procedure, it could only be employed when the focus point M was accessible, and it could never be used when M was a "virtual" focus situated behind the patient's eye, as in the case of an hypermetropic eye with accommodation relaxed. As the method of skiascopy is actually used, therefore, the interval between patient and observer is generally some convenient fixed distance, and instead of varying this distance the examination is conducted by regulating the position of the focus plane until there is no movement of the "shadow." The simplest mode of doing this is to insert a trial-case lens directly in front of the patient's eye, selecting first one lens and then another until one is found that neutralises the movement of the "shadow." In this case the optical system is to be regarded as the combination of the patient's eye and the interposed lens. In the method of *static skiametry*, where the accommodation of the patient's eye is supposed to be completely relaxed, a convenient working distance between patient and observer that is often used in this case is the fixed interval of exactly one meter.

Thus when a lens of refracting power F is inserted in the trial-frame in front of the patient's eye, the focus plane will be the plane that is conjugate to the retina with respect to the optical combination of eye and lens. If the lens could be removed without changing the state of refraction of the eye itself, the latter would be focused for some point X on the optical axis, but with the lens actually in place the entire optical system is focused for the point M. In other words, X and M are a pair of conjugate points with reference to the lens by itself. On the supposition that the lens is a thin lens with its optical center at a point L not far from the center A of the pupil of the eye, then by the ordinary lens-formula:

$$\frac{1}{\mathrm{LM}} = \frac{1}{\mathrm{LX}} + F.$$

Put $u=\mathrm{MA}$ and $U=1/u$; also $v=\mathrm{XA}$ and $V=1/v$, where V denotes therefore the state of refraction of the eye itself. Then if $c=\mathrm{AL}$ denotes the distance of the lens from the eye,

$$\mathrm{LM} = -\frac{1+cU}{U}, \quad \mathrm{LR} = -\frac{1+cV}{V};$$

and, consequently, after substituting these expressions in the lens-formula above and solving the equation for V, we shall obtain:

$$V = \frac{U+(1+cU)F}{1-c(1+cU)F},$$

which enables us to calculate the value of V in terms of that of U, provided we know also the magnitudes denoted by c and F. Usually, the interval c between the lens and the eye is comparatively so minute that it can be neglected entirely, and under such circumstances the formula reduces simply to $V=U+F$, $(c=0)$. For example, if the working distance is one meter ($\mathrm{AC}=1$ meter), then when there is no movement of the "shadow," and M coincides with C ($\mathrm{MA}=-1$ meter), we shall have therefore $U=-1$ dioptry and hence $V=F-1$; which is equivalent to the "rule for static skiametry at the distance of one meter," namely, that the

patient's static refraction is found by subtracting 1 from the power of the lens that neutralises the movement of the "shadow."

Formulas for the size of the "reflex circle" and the rate of movement of the reflex.—The distance of the mirror from the pupillary plane of the patient's eye may be denoted by z, that is, $z = AC$, and if the direction from A to C is taken as the *positive direction along the optical axis*, then the sign of z will always be positive. The point O is the foot of the perpendicular let fall from the center S of the image of the luminous source in the plane mirror on the optical axis; and if we put $CO = \epsilon z$, where ϵ denotes a positive number that ordinarily does not differ much from unity, although its actual value will depend on the distance between the lamp and the mirror, then $AO = (1+\epsilon)z$. The letters D and E, as previously stated, are used to designate the two opposite ends of the diameter of the pupil in the meridian plane, so that if the length of this diameter is denoted by $2p$, then $p = DA = AE$, where p is considered as being positive in sign. Let the diameter of the circular source of light be denoted by $2np$, where n is a positive number which may be greater than 2 and is nearly always greater than 1. (For example, an average diameter of the pupil is 4 mm and the diameter of the source of light is frequently as much as 10 mm, so that for this case the value of n will be 2.5.) The diameter of the image of the source of light in the plane mirror will likewise be equal to $2np$; that is, $S_1S_2 = 2S_1S = 2SS_2 = 2np$.

If, therefore, the straight lines SA and S_1E (Fig. 4) meet the focus plane in the points G and F, respectively, obviously

$$\frac{GF - AE}{MA} = \frac{S_1S + AE}{AO};$$

and, accordingly, after substituting the symbols $p = AE$, $np = S_1S$, $(1+\epsilon)z = AO$, and $1/U = u = MA$, and solving the preceding equation for GF, we shall obtain:

$$GF = p\,\frac{n+1+(1+\epsilon)zU}{(1+\epsilon)zU}.$$

The points designated by K and J are the central projections of the points G and F, respectively, on the pupillary plane from C as the center of projection, and evidently

$$\frac{KJ}{GF} = \frac{AC}{MC},$$

where K designates the center and $KJ = r$ denotes the radius of the

"reflex circle." The ratio AC : MC is the coëfficient of projection, which may be denoted by C; that is

$$r = C \cdot \mathrm{GF},$$

where

$$C = \frac{zU}{1+zU}.$$

Consequently we derive the following formula for calculating the radius of the "reflex circle":

$$r = p\,\frac{n+1+(1+\epsilon)zU}{(1+\epsilon)(1+zU)}. \qquad (1)$$

For example, suppose we assume that $\epsilon = 1$ and $n = 2.5$; then for these values the formula above becomes:

$$r = \frac{(7+4zU)\,p}{4(1+zU)}.$$

The corresponding values of r for various values of zU are exhibited in the following table:

zU	r	zU	r	zU	r
0	$\frac{7}{4}p$	−20	$\frac{73}{76}p$	−1.75	0
+1	$\frac{11}{8}p$	−10	$\frac{11}{12}p$	−1.50	$-\frac{1}{2}p$
+2	$\frac{5}{4}p$	− 6	$\frac{17}{20}p$	−1.25	$-2p$
+3	$\frac{19}{16}p$	− 3	$\frac{5}{8}p$	−1.00	∞
+5	$\frac{9}{8}p$	−2.5	$\frac{1}{2}p$	−0.75	$4p$
+10	$\frac{47}{44}p$	−2.	$\frac{1}{4}p$	−0.50	$\frac{5}{2}p$

Thus in the case of the unaided eye with accommodation completely relaxed, the "reflex circle" will be larger than the pupil, provided the eye is either emmetropic or hypermetropic. For very mild degrees of myopia from the lowest to a certain value of zU intermediate between -1 and ∞, the "reflex circle" will also be larger than the pupil, and for $zU = -1$ the size of the "reflex circle" will be infinite. On the other hand, for very high degrees of myopia, the "reflex circle" will be smaller than the pupil and will continually diminish in size as the myopia becomes less and less until for a certain critical value the "reflex circle" collapses into a point.

The ordinate of the center K (Fig. 4) of the "reflex circle" for a given position of the center S of the image in the mirror is

$$\mathrm{AK} = C \cdot \mathrm{MG};$$

and since

$$MG = \frac{MA}{OA} OS ,$$

it follows that

$$AK = C \frac{MA}{OA} OS .$$

Accordingly, substituting the expression found above for the projection coëfficient C, and putting $MA = u = 1/U$, $OA = -(1+\epsilon)z$, and $OS = y$, the following formula will be obtained for the ordinate of the center of the "reflex circle" corresponding to any given value of y:

$$AK = -\frac{y}{(1+\epsilon)(1+zU)} . \qquad (2)$$

If (disregarding the so-called "ragged edge" of the field of view and assuming that there is complete illumination to the extreme edge) the visible reflex phenomenon can be said to begin at the instant when the "reflex circle" touches the edge of the pupil, that is, when these two circles are tangent externally, this will be the case when the lower edge of the "reflex circle" is in contact with the upper edge of the pupil (or *vice versa*), that is, when the point L in Fig. 4 is coincident with the point E; which will be the case when $AK = AE + KJ = p + r$. If the straight line CE (Fig. 8) is drawn meeting the focus plane in the point

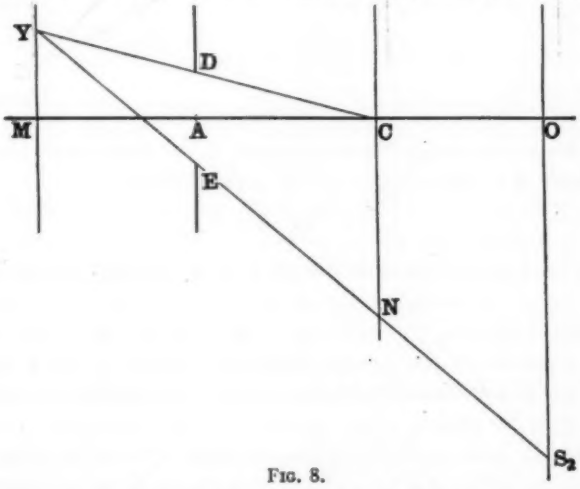

FIG. 8.

Y, the extreme position of the upper end S_2 of the diameter of the image in the mirror will be found by drawing the straight line YD meeting

the perpendicular to the optical axis drawn through O in the required point S_2.

This particular position of S_2 for which the lowest point of the "reflex circle" will be in contact with the highest point E of the pupil will be found to be such that

$$OS_2 = -p\{1+(1+\epsilon)(1+2zU)\};$$

and in order that the observer may see the entire reflex phenomenon from the moment when the two circles are tangent externally, obviously the mirror itself must extend at least as far below the axis as to the

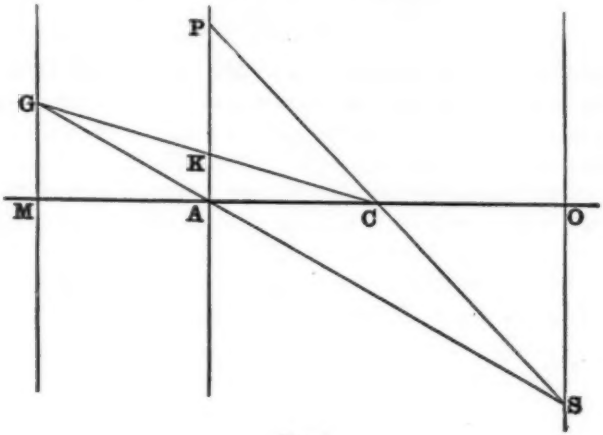

FIG. 9.

point N where the straight line YD meets the perpendicular to the axis drawn through C; the ordinate of this point N being

$$CN = -2p(1+zU).$$

Unless the *diameter of the mirror* itself is large enough, it may happen, therefore, that the reflex phenomenon will not begin to be visible at the instant when the "reflex circle" is tangent to the pupil, because the visible portion of the former will be that part of it that is common not only to the pupil itself but to the spot of light that is reflected from the mirror on the patient's face (considered here as being in the pupillary plane); in other words by taking account of the dimensions of the mirror another factor will be introduced in the problem, and the visible phenomenon will be found to depend on the intersections of three circles instead of being simply dependent on the two circles heretofore con-

sidered. However, it would lengthen this paper too much to study that question here, especially as it is not of any particular importance so far as the essential theory of skiascopy is concerned, because the form and size of the reflex is entirely secondary as compared with its *rate and direction of movement*, which we must proceed now to investigate.

The geometrical center of the spot of light reflected from the mirror on the pupillary plane (which, as already stated, may be considered as practically the same as the spot of light on the patient's face) will be at the point P (Fig. 9) where the straight line SC meets this plane; and as the center S of the image in the mirror moves from S to O, the spot of light on the face will traverse a corresponding interval equal to PA, and in the same time the center K of the reflex will pass over the interval KA. Accordingly, the speed of the two movements will be in the same ratio as KA : PA. Now it is evident from the diagram that

$$\frac{OS}{PA} = \frac{CO}{AC}, \quad \frac{KA}{OS} = \frac{MA}{MC} \cdot \frac{AC}{AO};$$

and, consequently,

$$\frac{KA}{PA} = \frac{CO}{AO} \cdot \frac{MA}{MC}.$$

Now

$$\frac{CO}{AO} = \frac{\epsilon}{1+\epsilon},$$

and if we put

$$\alpha = \frac{\epsilon}{1+\epsilon} v, \tag{3}$$

where v denotes the velocity of the movement of the spot of light on the patient's face, then the velocity (S) of the movement of the reflex will be given by the following formula:

$$S = \alpha \frac{MA}{MC};$$

which in terms of the symbols previously used may also be written:

$$S = \frac{\alpha}{1+zU}. \tag{4}$$

Accordingly, other things being equal, the speed of the reflex will be inversely proportional to the distance between the observer's eye and

the focus point of the patient's eye. As long as the sign of $(1+zU)$ is positive, that is, provided the focus point M is not situated between patient and observer, the sign of S will likewise be positive and the movement of the reflex will be in the same direction as that of the spot of light on the patient's face (or "with the mirror"); whereas when the focus point M is between A and C, the movement will be "against the mirror." The so-called neutral point or point of reversal is at C, and when the patient's eye is focused on C($zU=-1$), the speed of the reflex becomes infinite and there will be no apparent movement of the reflex. Evidently, the velocities of the reflex and the spot of light will be exactly the same when the focus point M coincides with the point C; and the two velocities will be equal and opposite for a certain position of M between A and C such that

$$AM=\frac{1+\epsilon}{1+2\epsilon}AC.$$

When the focus point M is near the neutral point C, this parallax method will be more sensitive for a greater interval between the two eyes than for a shorter interval.

Criticism of the Method of Skiascopy for Measurement of Refraction.— It is perhaps scarcely necessary to add that in drawing conclusions from the geometrical theory as outlined above, its mathematical limitations must be kept steadily in mind, and above all the assumption that the optical system is free from aberrations along the axis and that consequently there is strict point-to-point correspondence between object and image, which is never absolutely the case in any real dioptric system, much less in the complicated optical apparatus of the human eye. The most that can be said, therefore, is that the results which we have obtained will be more reliable in proportion as this condition of freedom from aberrations is satisfied, which in turn will depend to a large extent on the actual size of the pupil during the examination of the patient's eye. The concomitant effects of the illumination of the eye are not only a contraction of the pupil but at the same time a very decided tendency to blind the eye so far as distinct foveal vision is concerned. Moreover, accommodation is likewise accompanied by contraction of the pupil. A necessary condition in the method of *static skiametry* is that the accommodation shall be completely relaxed, and accordingly in actual practice oculists are in the habit of using a cycloplegic drug the temporary effect of which is to paralyse the muscles of accommodation; but all these drugs are also mydriatic and

produce at the same time a temporary enlargement of the pupil, which introduces new factors in the problem and renders the determination of the refraction of the eye by this process very uncertain, to say the least, particularly in regard to the so-called optical zone of the pupil.[2] In so-called *dynamic skiametry*, on the contrary, the accommodation of the eye is not relaxed, and the patient is required to fixate very exactly a target of some kind and to decipher clearly the letters or characters exposed on it. However, the difficulty in this case is that owing to the reduced foveal sensitivity in consequence of the illumination of the fundus of the eye, the patient is not able to focus distinctly; and so in the method of dynamic skiametry as commonly employed, the patient fixates the target with his other eye, the assumption being that then both eyes are focused on the same point. However, this introduces a new factor entirely and raises questions connected with binocular convergence and the relative amplitude of accommodation for a given degree of convergence, etc.

The practical trouble in all these cases is that one can never be absolutely sure as to how the patient's eye is really focused at the moment when the measurement is being made. In the case of a fixed optical system consisting, for example, of a single convex lens of such small relative aperture that the aberrations along the axis are practically negligible, it is a comparatively simple matter to find the point of reversal with a skiascope and thus to determine with considerable precision the focus plane conjugate to a certain transversal plane on the other side of the lens beyond its focal point; but the problem is by no means so easy when the optical system is an *active living eye* capable of a great variety of unconscious adjustments and when, moreover, the luminous screen behind the optical apparatus is the fundus of the eye illuminated from in front by light that has first to be transmitted through the optical system itself. Thus while theoretically it should be possible according to formula (4) to find the precise position of the neutral point for which there is no movement of the reflex, nevertheless in actual practice, especially when the distance between the patient and the observer is less, say, than half a meter, instead of locating a perfectly definite point, we find rather a certain limited *neutral region* situated, it is true, in close proximity to the mirror, and more in evidence at close range than when the two eyes are farther apart, which will also be larger in extent with increase in the size of the pupil of the

[2] See English trans. of Helmholtz's *Physiological Optics*, I, p. 429.

patient's eye. In such a case all that can be definitely stated is that the focus point is somewhere within this region without being able to say exactly where it is or even whether it is in front of the mirror or behind it.

To take a specific instance, let us suppose that the examination is being purposely conducted at a distance not much greater than the average normal distance of distinct vision for near work, say 13 inches (or about one third of a meter), and that the patient is required to fixate a target at this distance and to distinguish clearly certain minute details on it. Under such conditions it is practically invariably the case that the skiascope will indicate a decided movement "with the mirror," and in order to find the point of reversal the distance between the two eyes has to be slightly increased without altering any of the other conditions. Various hypotheses, more or less ingenious, have been offered to account for the neutral region and to explain the apparent discrepancy between theory and practice which has always been something of a stumbling block in the way of dynamic skiametry. Thus, for example, it has been suggested with much show of reason that the difficulty that is encountered here in this particular instance is due to a certain lag of accommodation behind the convergence of the two eyes, so that the measurement would really amount to a determination of one portion of the amplitude of relative accommodation for a given degree of convergence. The trouble with this explanation is that exactly the same result is obtained with the skiascope when the illumination is dim and the patient fixates with the illuminated eye, while his other eye is completely screened.

The writer is persuaded that the true explanation of the whole matter is to be sought in the *monochromatic aberrations of the human eye*, and that, admirable and useful as the ordinary method of skiascopy is for the determination of refraction, it should always be checked and supplemented by subjective methods of measurement. Gullstrand's mathematical and stigmatoscopic investigations of the complicated form of the caustic surfaces in the case of a bundle of rays that have traversed the dioptric system of the eye (as described, for example, in Chapter V of the Appendix of Volume I of the English Translation of Helmholtz's *Physiological Optics*[3]) show that with a pupil of moderate

[3] See also A. Gullstrand, *Einführung in die Methoden der Dioptrik des Auges des Menschen* (Leipzig, 1911), which is a work that should be carefully consulted by all ophthalmologists and by oculists and optometrists who are really scientific refractionists. Incidentally, this treatise contains a thorough description of the author's methods of both subjective and objective stigmatoscopy, besides a critical survey of the various optometrical methods of refraction.

size the place or section of most advantageous convergence of the bundle of rays for distinct vision is quite appreciably different from the cusp of the caustic surface where the so-called paraxial rays are focused. In fact his measurements indicate a difference of focus on account of aberration which in the case of an emmetropic eye with pupil of moderate size may amount to as much as 1.5 dioptries of hypermetropia along the axis. Consequently, in his very careful calculation of the schematic eye, Gullstrand has compensated for the effect of aberration by allowing a certain average value of one dioptry; in other words, the refraction of the unaccommodated schematic eye must be equal to one dioptry of hypermetropia in order that it shall be equivalent to an actual emmetropic eye under the same conditions. Now in the optical theory of skiascopy as developed above no account whatever was taken of the effect of aberration. The so-called *focus plane* was defined simply as the plane conjugate to the surface of the retina; which would be identical with the transversal plane conjugate to the cusp of the meridian section of the caustic surface, if the cusp itself were situated on the retina. But since this cusp is not after all the most advantageous place of convergence of the rays for distinct vision, the *focus plane will not coincide with the plane on which the eye is fixated*, that is, the plane of the target on which the patient is able to see the details distinctly. When the mirror is adjusted in the latter plane, it is natural to expect, therefore, that there will be a movement of the reflex exactly as is found to be the case. When the eye is accommodated for comparatively near vision, the change of focusing required to shift the retina, figuratively speaking, from one section of the bundle of refracted rays to another closely adjacent section may easily amount to as much as a dioptry or more. Consequently, the whole reason for the confusion that has been the subject of so much controversy especially in connection with dynamic skiametry and for the apparent discrepancy between theory and practice would seem to be due to the unjustifiable identification of the so-called focus plane with the plane of the actual target on which the eye is fixated.

Let the point where the optical axis of the eye meets the surface of the retina be designated by B and let the point where it meets a certain section of the bundle of rays closely adjacent to the retina be designated by N; and put $BN=e$, where the interval e, supposed to be measured in millimeters, is reckoned positive or negative according as N is taken beyond B or between B and the crystalline lens, respectively. If A denotes the static refraction of the eye, that is, its refraction when the accommodation is entirely relaxed (in which case the so-called focus point will be identical with the far point of the eye), the change of focusing that will be necessary in order to shift the section at N on to the retina at B can be calculated from the following formula:

$$\Delta A = -\frac{(A+F)^2}{A+F+1000n'/e},$$

where F denotes the refracting power of the eye and n' is the value of the index of refraction of the vitreous humor. Employing the data of Gullstrand's schematic eye, namely, $F = 58.64$ dptr and $n' = 1.336$, we can compute the value of ΔA corresponding to any given pair of values of A and e. Thus, for example, for $A = 0$ (emmetropic eye) and $e = -1$ mm, we find $\Delta A = +2.69$ dptr; and for $A = -3$ dptr (myopic eye) and $e = -0.5$ mm, we find $\Delta A = +1.18$ dptr.

The conclusion is, therefore, that while ordinary skiascopy is a clinical method of the greatest usefulness and convenience for finding the state of refraction of the eye, the result needs to be checked by other methods when a high degree of accuracy is required. The important question after all is as to how the individual patient actually focuses his eye for distinct vision, and so far as clinical methods are concerned, the ultimate decision here will probably have to depend on the subjective investigation. In any case the discussion seems to indicate beyond any doubt that the monochromatic aberrations are a more important factor than has usually been supposed; and possibly a careful consideration of this element in the problem of refraction might shed light on some of the curious idiosyncrasies of vision that are reported by oculists and optometrists in their ordinary practice. The greater the interval between patient and observer, the more sensitive the method of skiascopy proves to be, and at the same time the observer will not have to be so careful about maintaining this distance exactly. On the other hand, for convenience of manipulation, the patient should be within arm's reach of the person who is conducting the examination; and so, on the whole, Gullstrand recommends a working distance of 50 cm. It is scarcely worth trying to obtain any higher degree of accuracy than can be obtained at this distance, for it is likely to be more apparent than real. The refraction in the *fovea centralis* itself cannot be found by ordinary skiascopy, and for this purpose it will be necessary to employ a slight modification of Gullstrand's method of *objective stigmatoscopy* and to use a transparent mirror in conjunction with a Nernst slit-lamp or some other source of illumination of equal specific intensity.

In the preparation of this paper the writer has received a number of valuable suggestions from his colleague, Mr. C. L. Treleaven, who is exceedingly expert in the use of the skiascope. He is also indebted to Mr. Treleaven for drawing the geometrical diagrams.

DEPARTMENT OF PHYSICS,
COLUMBIA UNIVERSITY, NEW YORK CITY,
MARCH 1, 1926.

SPECTRAL FILTERS[1]

By K. S. Gibson

INTRODUCTION

Many substances, such as glass, crystals, dyes, solutions, etc., exhibit the well known property of absorbing certain regions of the spectrum while transmitting other parts more freely. Such selective absorption in the visible gives rise to the sensation of color. When for any reason in spectral analysis it is not feasible to isolate the desired regions by means of the spectrometer, filters may often be successfully used. Their advantages consist in their simplicity and conservation of energy. They do not, of course, afford the fine analysis possible with the spectrometer.

It is not the purpose of the present article to catalogue or illustrate the uses to which such filters may be put. In a paper by Potapenko, translated into English by Mees (36)[2], a very extensive bibliography is given, especially of the foreign literature; and the author states, "The use of light-filters for all scientific purposes has increased greatly of late years. Physicists, astronomers, chemists, doctors, all make use of them to obtain monochromatic rays of light. Botanists employ them to study the influence of light of various colours on plant life, ... and it may be said with full justification that at the present time there is no field where light-filters are not employed." Recent illustrations of the varied uses for spectral filters may be noted in papers by Coblentz, who has used them in his studies of the photoelectric properties of molybdenite (7), the energy distribution in the spectra of stars (9), and the germicidal action of ultraviolet radiant energy (10).

It is attempted rather to give herein a compilation of data which may be of value to those contemplating the use of filters. The paper should fill the needs of those who ask the Bureau of Standards for such information. Its scope is limited to those spectral regions commonly designated as the ultraviolet, visible, and infrared, thus excluding x-rays on the one end and electric waves on the other. Furthermore, the special filters used for photometry, photography, and pyrometry may

[1] Published by permission of the Director, Bureau of Standards.

[2] Numbers in parentheses, followed occasionally by page or figure references, refer to the Bibliography at the end of the paper.

be found described in the literature on those subjects and will not be particularly noted here.

The data given below, while based in most cases on quantitative measurements, are to be considered as illustrative only, and no attempt is made to indicate their reliability. This is because the authors have failed either to state the reliability of their data or to give the purity or analysis of the material, or because of the variability in the material itself. It is believed, nevertheless, that investigators will be interested in thus having illustrated to some extent what is available for transmitting or absorbing the various spectral regions.

Because of their convenience and permanence, glasses and crystalline material have been mostly chosen for illustration. Complete data, infrared as well as visible and ultraviolet, are also more generally available for these substances. In some cases, as may be noted, data for other materials are given, particularly where similar data for glasses or crystals do not exist. As a rule, simple well known substances only are noted. It is, of course, realized that much of the data presented herewith between 300 and 700 mμ may be practically duplicated by means of dyes, inorganic salt solutions, or other substances, but except in the case of the Wratten dyed gelatine filters (45) quantitative data are largely lacking or widely scattered throughout the literature.

Three classes of filters may be noted: (1) those transmitting freely at longer wave lengths and absorbing strongly at shorter wave lengths, (2) those transmitting freely at shorter wave lengths and absorbing strongly at longer wave lengths, and (3) those transmitting a narrow region of the spectrum while strongly absorbing other regions. Class 3 may obviously also be obtained by the proper combination of classes 1 and 2. As a rule, only those filters are considered which have a relatively sharp transition between the regions of free transmission and strong absorption.

So far as possible the filters have been treated as if they were to be used with a non-selective detector of radiant energy such as the thermopile. This is the general case, and applies also to investigations of the effect of energy of different wave lengths on living organisms, the fading of dyes, etc. If the photoelectric cell (electron emission) or the ordinary photographic plate is the detector, the infrared region need not, in general, be considered. If visual work is contemplated the problem is still simpler, as the ultraviolet region may also be disregarded, except from the standpoint of eye protection. The emission characteristics

of the sources of energy used with the filters should, of course, likewise be known, as well as any other qualifying circumstances.

THE LAWS OF TRANSMISSION

Certain definitions and the statement of certain relations seem necessary to a proper understanding and use of transmissive data[3]:

1. Rectilinear transmission of homogeneous radiant energy through a homogeneous medium of thickness b, with plane, parallel, polished surfaces.

Let: $E_1 \equiv$ radiant energy incident on the first surface.
$E' \equiv$ radiant energy transmitted by the first surface.
$E_2 \equiv$ radiant energy incident on the second surface.
$E'' \equiv$ radiant energy transmitted by the second surface.

Then: $T \equiv E''/E_1 \equiv$ transmission.
$\mathsf{T} \equiv E_2/E' \equiv$ transmittance.
$t = \sqrt[b]{\mathsf{T}} \equiv$ transmissivity
$\log \mathsf{T} = b \log t$ } Lambert's law.

2. Rectilinear transmission of homogeneous radiant energy through a cell containing a homogeneous solution of thickness b and concentration c.

Let: $T_{sol} \equiv$ transmission of a cell containing the solution
and $T_{solv} \equiv$ transmission of the same or a duplicate cell containing solvent only.

Then: $\mathsf{T} \equiv T_{sol}/T_{solv} \equiv$ transmittancy.
$\mathbf{t} = \sqrt[bc]{\mathsf{T}} \equiv$ specific transmissivity
$\log \mathsf{T} = bc \log \mathbf{t}$ } Beer's law.

Beer's law is often only approximate.

In Figs. 1 and 2 curves stopping at 0.75 represent transmissions, those extending above this value, transmittances or transmittancies. All the curves of Figs. 3a, 3b, and 4 are transmission curves (except those labeled *Cl* and *Cl-Br*, which values represent probably the ratios of the transmission of a cell filled with the gas to that of the same or a similar cell either filled with air or evacuated).

[3] Further discussion of these relations is given in references 16, 17, 19, 23, 33, and 36 of the Bibliography.

ILLUSTRATIVE DATA

Class 1. *Filters Absorbing Strongly at All Shorter Wave Lengths While Transmitting Freely at All Longer Wave Lengths into the Infrared.*

For the purposes of this article, the ultraviolet will not be considered for the wave length region below 120 mμ. No known solid or liquid of appreciable thickness transmits within this region until the x-ray spectrum is reached. The transmission limit of 1 cm of dry air at atmospheric pressure is near 175 mμ (27, p. 24), and most experimental work below about 185 mμ has had to be done with vacuum spectrographs. Of the ordinary gases, hydrogen is the most transparent,

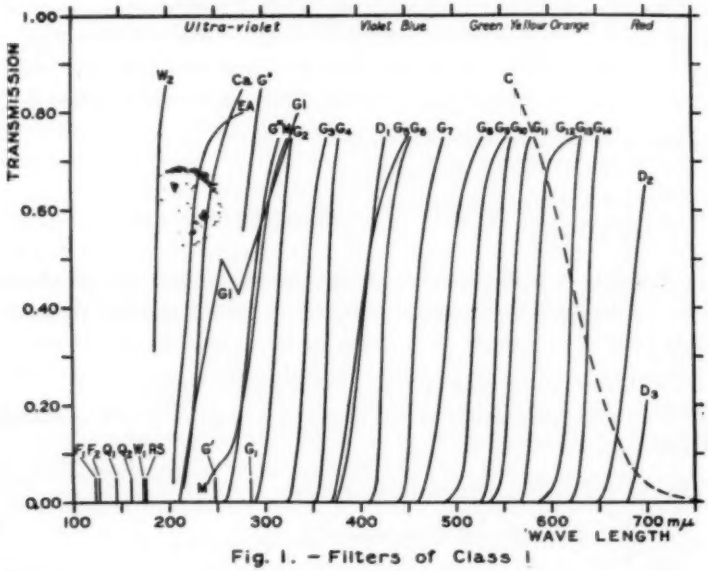

Fig. 1. – Filters of Class 1

oxygen the most absorbing for this range from 120 to 185 mμ (27, pp. 61–73).

In Fig. 1 are given illustrative data on filters of class 1, with explanatory remarks and references in the legend. Data for the transition regions only are shown, or in some cases as indicated by the short vertical lines, only the "limits" of transmission. All these filters transmit freely but often far from uniformly at all longer wave lengths into the infrared. The regions where the filters again absorb strongly may be noted from the legend.

It is apparent that by properly choosing the thickness or concentration, curves similar to those of Fig. 1 may be obtained at practically

LEGEND TO FIG. 1. *Filters of Class 1.*

Filter (Curve) No.	Material	Remarks	Thickness in Millimeters	Reference				Infrared Limit
				Bibliography No.	Page	Fig.	No.	
F_1	Fluorite	Best quality, colorless	Thin plates	27	75			Cf. Fig. 2.
F_2	Fluorite	Schumann's map	Prism+lenses	27	75			Cf. Fig. 2.
Q_1	Quartz	Crystalline	0.2	27	56			Cf. Fig. 2.
Q_2	Quartz	Crystalline. No great difference for right or left handed pieces or for those cut $\perp$or $\parallel$ to axis	2	27	56–57, 13			Cf. Fig. 2.
W_1	Water	Distilled—fluorite windows —prolonged exposure	0.5	27	60			Cf. Fig. 2.
W_2	Water	Pure—quartz windows	?20	25 27	 60			Cf. Fig. 2.
RS	Rock Salt		?	27	57, 13–14			Cf. Fig. 2.
EA	Ethyl Alcohol	Chemically pure, from Merck	10	34				Approximately equivalent to water.
Ca	Calcite	$\perp$ to axis	6.1	34				Cf. Ref. 31.
Gl	Glycerine	Chemically pure, from Merck	10	34				Approximately equivalent to water. Cf. Ref. 8, Fig. 5.
M	Mica		0.01	10	653	4	2	Cf. Ref. 3, No. 65, pp. 48–49.
G′	Glass	Jena "Uviol" cover glass	Extremely thin	27	12			So far as known, all white, yellow, orange, and red glasses have infrared transmissions similar to G_1–G_2 of Fig. 2. Cf. also Fig. 3b. Those containing iron impurities have a broad weak absorption band centering at 1.1μ. Cf. Ref. 6 and 11.
G″	Glass	Jena "UV Crown 3199"	1	21 27	7 11–12			
G‴	Glass	Corning "G 174 S Ultra"	0.39	17		3	4	
G_1	Glass	Common cover glass	Extremely thin	27	12			Cf. above.
G_2	Glass	Am. Opt. Co., "Crown 1.50"	1.68	16 17		2 2	A 1	
G_3	Glass	Corning "G 111 CD"	8.30	17		2	2	
G_4	Glass	Am. Opt. Co. "Crookes A" (or "Cruxite")	2.05	16		3	A	
G_5–G_8	Glass	Corning "Noviol"	2.00–4.23	16 17		8 7	0 23–25	Cf. above.
G_9–G_{12}	Glass	Corning "G 34"	1.50–3.45	17 17		7 8	26, 29, 30 31	
G_{13}	Glass	Corning "G 24" ("Signal Red")	0.88	17		6	18	
G_{14}	Glass	Jena "4512"	2.25	19	191	5		
D_1–D_3	Dyed Gelatine films	E. K. Co. Wratten filters. Data not given above 700 mμ. Most of curves G_5–G_{14} can be practically duplicated by the Wratten filters.	?	45			2, 89, 89*a*	Cf. Ref. 6, p. 655, and Ref. 28
C	Aqueous solution of copper sulphate	57.0 grams of $CuSO_4.5H_2O$ to 1 liter of solution. Filter of Class 2. Transmits ultraviolet to about 300 mμ.	10.00	18	116	1		Cf. Ref. 5.

any wave length from the extreme ultraviolet to the infrared. A series of curves of this type between 200 and 350 mμ may doubtless also be obtained by the use of certain organic solvents (1; 2; 42), and from 350 to 700 by the use of dyed gelatine films (45), dye solutions, or inorganic solutions. Filters transmitting only (a portion of) the infrared (with a trace of the far red) are illustrated by curve D_2 of this figure and by curve G_9 of Fig. 3b. A series of curves in the infrared somewhat analogous to those of Fig. 1 may also be obtained (41, p. 523; 3, No. 97, p. 46; 38, p. 1132) by the use of thin layers of lampblack or various thicknesses of black paper or cardboard.

Class 2. *Filters Absorbing Strongly at Longer Wave Lengths While Transmitting Freely at All Shorter Wave Lengths into the Ultraviolet.*

The data of this section, Fig. 2, must be qualified by reference to the information regarding transmission at still longer wave lengths in the infrared, below. The ultraviolet limits of transmission are indicated.

Curve *C* of Fig. 1 continues the series of filters of class 2 into the visible. Aqueous solutions of the copper salts are the only filters available for completely absorbing the infrared while transmitting so freely at the shorter wave lengths.[4] It is best to use two or more centimeters of solution, reducing the concentration if desired, to insure the complete removal of the infrared (5). Blue-green glasses are available (14; 16; 17) for visually approximating curve *C*, and for extending the filters of class 2 farther into the visible, but all transmit more or less in the infrared (6; 11).

Class 3. *Filters Transmitting at More or Less Narrow Spectral Regions While Absorbing Strongly at Other Regions.*

These filters, Figs. 3a and 3b, may be used alone or in combination with classes 1 or 2, dependent upon circumstances. It is because filters of this class are somewhat limited that the use of classes 1 and 2 may be necessary. The following points may be noted:

(1) There seems to be no filter or combination of filters available for transmitting only the region below 300 mμ in whole or in part. The nearest approach to such a filter consists of quartz cells filled with chlorine gas and bromine vapor (33), chlorine having a maximum absorption at approximately 335 mμ, bromine at 430 mμ, and both transmitting freely below 300 mμ. Data (circles) for chlorine alone and for chlorine plus bromine, under the experimental conditions noted in the

[4] Dr. H. P. Gage of the Corning Glass Works kindly informs the writer that a water cell in combination with Corning glass G 392 H will absorb all the infrared. Reference to Dr. Gage's paper (Ref. 14 of the Bibliography) will furnish considerable information of value in the use of colored glasses as filters.

legend, are given in Fig. 3a[5]. Other references to the transmission of chlorine (32; 40) and bromine (44) may be noted. Aqueous solutions of acetone (1) and paranitrosodimethylaniline (33; 39, Fig. 3) and doubtless many other substances may also assist in ultraviolet isolation,

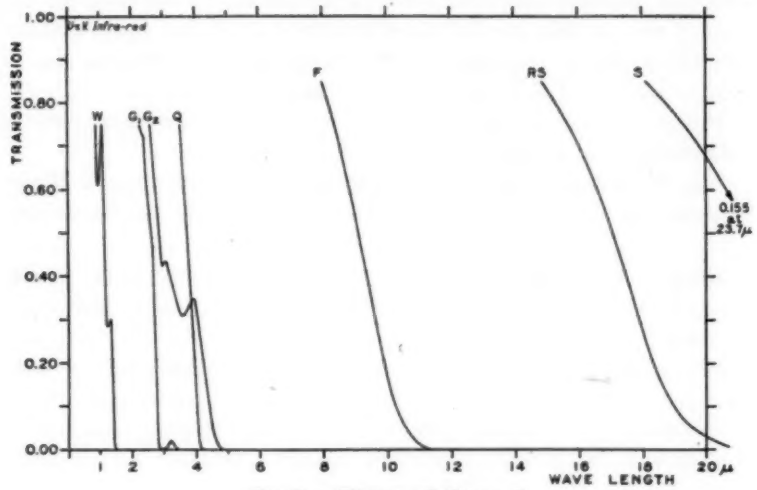

Fig. 2. – Filters of Class 2

especially where the ordinary photographic plate, insensitive above 500 mμ, is used as the detector.

(2) Between 300 and 550 mμ (in combination with class 1 if neces-

[5] The data given in Fig. 3a represent perhaps the most efficient experimental conditions for isolating the region below 300 mμ. The author also gives the following values of "percentage of light transmitted" through a mixture of chlorine and bromine which "comes near to concentrations of Cl and Br at normal conditions of temperature (18°) and pressure (740 mm)," (Ref. 33, Eng. Trans., p. 398). The cell thickness was 10 mm and the molar concentrations 0.044 and 0.0046 for the chlorine and bromine, respectively.

Wave length	Percent Transmission
579.0	37.3
546.0	36.8
435.9	3.0
404.8	1.2
365.0	0.7
313.0	0.6
302.0	2.2
296.7	7.2
265.0	34.7
253.6	42.9
232.6	36.8
226.3	35.8

LEGEND TO FIG. 2. *Filters of Class 2.* (Cf. also Curve C, Fig. 1.)

Filter (Curve) No.	Material	Remarks	Thickness in Millimeters	Reference				Ultraviolet Limit
				Bibliography No.	Page	Fig.	No.	
W	Water	Thin quartz windows	10	9	734	3		Cf. Fig. 1.
G_1	Glass	Corning "Pyrex"	11.9	9	735	3		Cf. Ref. 17, Fig. 3, No. 6.
G_2	Glass	Crown	2.18	12	257	1		Cf. Fig. 1.
				11	8	2	E	
Q	Quartz	Crystalline	4.77	9	734	2		Cf. Fig. 1.
F	Fluorite	Colorless	10	37	737			Cf. Fig. 1.
RS	Rock Salt		10	37	737			Cf. Fig. 1.
S	Sylvite	Strong narrow absorption bands at 3.18 and 7.08 μ	10	37	737			
				4	653			

sary) more or less narrow transmission bands may be obtained centering at any desired wave length. Filter *C*, Fig. 1, may be combined with these if it is desired to remove the red and infrared.

(3) Transmission bands between 550 and 700 mμ may be obtained by combining filters of type *C*, Fig. 1, with the proper filters of class 1. Curve *C-K* of Fig. 3a illustrates this.

(4) A 1-cm cell of water, filter *W*, Fig. 2 (or glycerine, 8, p. 272, or 1.67 mm of alum, 31) in combination with filters G_3 or G_4, Figs. 3a and 3b, plus a red glass, or in combination with filter D_3, Fig. 1, will isolate the infrared from about 0.7 to 1.5 μ. Greater thicknesses of water will, of course, lower this upper limit.

(5) The infrared from approximately 1 to 3 μ may be isolated by a combination of filters G_{14} or D_3, Fig. 1, G_1, Fig. 2, and G_7, Fig. 3b; the region 2 to 3 μ by the same combination except with filter G_6, Fig. 3b, replacing filter G_7.

(6) In the infrared of longer wave lengths filters of this class may be approximated by the multiple selective reflection of energy (residual rays) from various crystals—centering near 8–9 μ in the case of quartz and many other substances (3) up to 152 μ for thallium iodide (41). Extrafocal methods may also be used to assist in spectral isolation both in the ultraviolet (13, p. 442) and infrared (38; 41).

Attention should perhaps be especially called to the extensive set of filters sold by the Eastman Kodak Co. under the name of the Wratten filters and described in a pamphlet under this title (45). In addition to various photographic and photometric filters, the list includes a large number which would come under the present classes 1 and 3. Information is given regarding their ultraviolet and visible transmission only, but it is probable that they all transmit more or less freely in the infrared. Most of the filters consist of dyed gelatine films, conveniently mounted if desired between glass plates and furnished in various sizes. Their degree of stability is noted. Here, as in the case of glass filters, spectral transmission measurements should be made on the actual filter used, if such exact quantitative information is of importance.

SPECIAL PROBLEMS

1. The separation of the ultraviolet, visible, and infrared.

Data already given serve to illustrate the feasibility of:

(1) Absorbing the ultraviolet while transmitting most of the visible and considerable of the infrared (class 1).

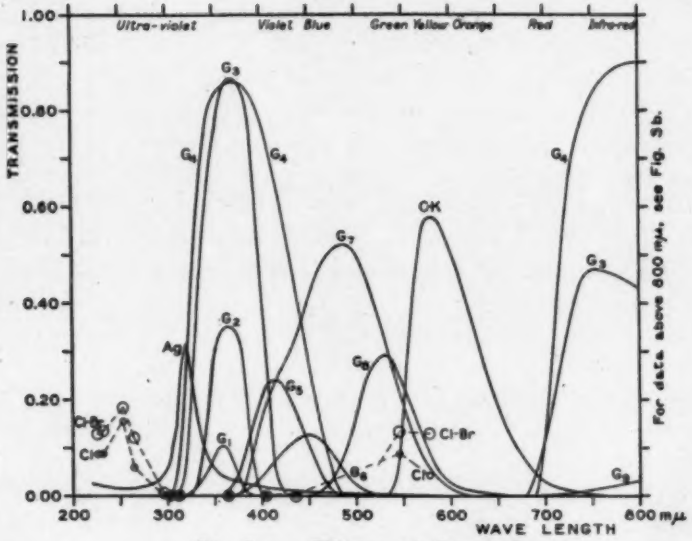

Fig. 3a. – Filters of Class 3

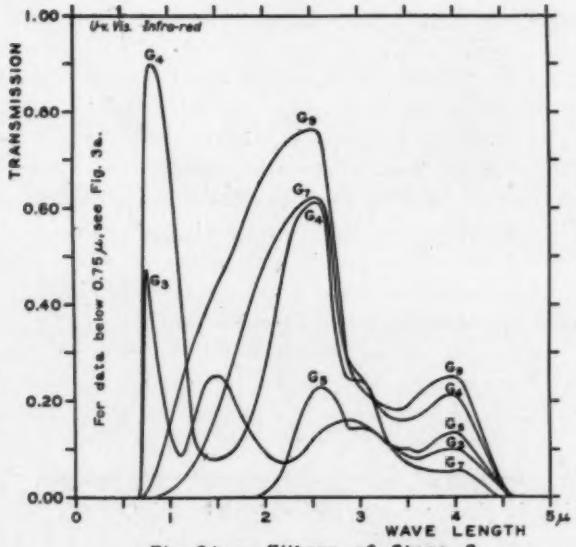

Fig. 3b. – Filters of Class 3

LEGEND TO FIGS. 3a and 3b. *Filters of Class 3.*

Filter (Curve) No.	Material	Remarks	Thickness in mm.		Reference					
					3a			3b		
			3a	3b	Bib. No.	Fig.	No.	Bib. No.	Fig.	No.
Cl	Chlorine gas	Quartz cell. Molar concentration of gas = 0.176.	10.		33	(Table 4)*				
Cl–Br	Chlorine gas and bromine vapor, mixture.	Quartz cell. Molar concentrations of chlorine and bromine = 0.088 and 0.0092, respectively. Advantage of combination over chlorine alone lies in the strong absorption in the blue where chlorine transmits.	10.		33	(Table 5)*				
Ag	Metallic silver	Chemically deposited on quartz. No infrared transmitted.	0.000079		20			20		
G_1	Glass	Corning "G 586 P." No infrared.	4.37		17 (Cf. 14)	21	87			
G_2	Glass	Corning "G 586 J." No infrared.	2.68	2.80	17 (Cf. 14)	21	85	6	(p. 667)	
G_3	Glass	Corning "G 586 A."	3.20	2.85	17 (Cf. 14)	21	83	6	14	C
G_4	Glass	Corning "G 585." (Equivalent to Jena "3654".)	4.62	3.13	17 (Cf. 14)	17	69	6	13	A
G_5	Glass	Corning "G 53."	2.59	2.40	17 (Cf. 14)	18	75	6	13	D
G_6	Glass	Jena "8780." Infrared somewhat similar to G_4.	2.99		17	15	63			
G_7	Glass	Corning "Signal Green."	5.55	4.93	17 (Cf. 14)	14	58	6	10	C
G_8	Glass	Jena "4930" (Equivalent to Corning Sextant Green.") Infrared similar to G_7 and G_9.	2.99		17 (Cf. 14)	12	51			
G_9	Glass	Am. Opt. Co. "Electric Smoke." Should be tested for possible violet transmission.		2.46	16 (Cf. 14)	21	K	6	14	B
C–K	Aqueous solutions of copper sulphate and potassium dichromate, combined transmission.	57.0 grams $CuSO_4.5H_2O$ to 1 liter of solution. 72.0 grams $K_2Cr_2O_7$ to 1 liter of solution. Practically no infrared, but Cf. Ref. 5.	10.0 mm of each solution in separate glass cells.		18	1	(Table 2)			

* These table numbers refer to English translation.

(2) Absorbing the ultraviolet and most or all of the visible while transmitting considerable of the infrared (class 1).

(3) Absorbing all of the infrared while transmitting most of the visible and the ultraviolet to about 300 mμ (filter *C*, Fig. 1).

(4) Absorbing all of the ultraviolet and infrared while transmitting most of the visible (combined filters G_6 and *C*, Fig. 1). A filter whose transmission closely approximates the visibility (spectral luminous efficiency) curve has been developed (Ives, H. E., The Photometric Scale. Jour. Frank. Inst., *188*, pp. 217–235; 1919).

(5) Absorbing all the infrared and visible while transmitting a certain part of the ultraviolet (filters G_1 and G_2, Fig. 3a. Note also filter *Ag*).

2. The isolation of spectral lines.

Because of their great intensity the mercury spectral lines, especially from the quartz arc, are of considerable value for research work. The strong lines in the visible are at 577+579 mμ (yellow), 546 (green), 436 (blue-violet) and 405+408 (violet). In the ultraviolet there is a group of intense lines at 365 mμ, other strong lines at 335, 313 and 303 mμ, and many others below 300, notably the one at 254 mμ. In the near infrared there is a group of lines between 1.0 and 1.8 μ, the strongest being at 1.014 and 1.129 μ. Energy of wave length approximately 300 μ is also radiated from the quartz mercury arc.

The visible lines may be readily isolated*—by glasses (17, p. 19), by solutions (24; 43, p. 15), or otherwise (45, p. 78). Filters of classes 1 or 2 may be combined with these if necessary, to absorb the ultraviolet or infrared. Filters *Cl* (32), *Cl-Br*, *Ag*, G_1 or G_2 (Fig. 3a), combined with filters of class 1 if desirable, will assist in the isolation of certain of the ultraviolet lines. The infrared lines may also be isolated (Cf. (4) under class 3, above) provided the weak continuous spectrum present in this region does not render this impracticable.

Certain visible lines from the helium and hydrogen vacuum tubes are readily isolated (17, p.19). Helium also has an intense line at 1.084 μ and a weak line at 2.026 μ. Most of the energy of the Bunsen flame is concentrated at 4.4 μ and may be isolated by a filter absorbing the ultraviolet and visible.

* All the filters for isolating the green and yellow mercury lines transmit freely in the red, and it may often be necessary to combine a blue-green glass (absorbing strongly in the red) with the filters noted in the references before satisfactory work can be done with these lines. This is especially true with the quartz-mercury arc which shows an appreciable red continuous spectrum in addition to the weak red lines.

3. *The absorption of the yellow mercury, sodium, copper, or helium lines.*

Glasses (17, Fig. 4) and solutions (39, Figs. 96 and 100) containing "didymium," of the proper thickness or concentration, exhibit a remarkably strong and sharp absorption in the spectral region 570–590 mμ while transmitting the rest of the visible and much of the ultraviolet and infrared quite freely. Such filters are useful in separating the mercury green and yellow lines (Cf. 2, above), in absorbing the sodium lines in flame analysis (14, p. 44), in helping to protect the eyes from the blinding glare of copper arcs (17, p. 118), and so on. Another filter for use in flame analysis may be noted (28).

4. *Stray light filters in spectrophotometry.*

With the usual prism instrument and incandescent sources increasing error may be introduced in spectrophotometric measurements below 500 mμ or above 650 mμ if stray light is disregarded (19, pp. 190–191). Efficient filters for its elimination are illustrated by some of the present data. For work in the blue and violet below 450 mμ, curve G_4, Fig. 3a, represents a very suitable filter, the far red band being negligible in its effect. Between 460 and 500 mμ, a less dense blue filter may be used if necessary. For work in the red, filters having transmissions of the type of curve G_{14}, Fig. 1, are very efficient. For the far red region, the two types of filter may be combined, or No. D_2 or D_3, Fig. 1, may be used.

5. *Trichromatic photometry.*

In certain visual problems sufficient spectral analysis may be obtained with a photometer and colored filters without recourse to a spectrophotometer. For this purpose a set of red, green and blue filters of the type represented by curve G_{14}, Fig. 1, and curves G_8 and G_5 (or G_4), Fig. 3a, will usually be desired.

6. *Eye-protection.*

In general, it may be said that radiant energy in the visible and infrared regions does not cause injury to the eye, provided that the conditions of brightness and temperature are such that the eye is experiencing no discomfort. Serious injury to the cornea may, however, be sustained by even short exposure to the ultraviolet below about 310 mμ.

Most glass of two or more mm is a sufficient protection against this region, as may be noted by reference to Fig. 1. To be on the safe side, however, especially if one is working around sources which are known to emit considerable energy in this harmful region, such as the quartz-mercury arc, and bare metallic arcs or sparks, one should protect the

eyes with filters completely absorbing all wave lengths below about 350 mμ.

Glasses are also available for reducing brightness to any desired degree or for absorbing the larger part of the infrared (11; 12; 16).

7. *The infrared of long wave lengths.*

The infrared spectral region has been extended experimentally to about 300 μ (0.3 mm). Such information as we possess beyond 20 μ is due principally to the work of Rubens and his collaborators. The methods used and much of the transmissive and reflective data thus obtained from 50 to 300 μ have been summarized by Weniger (41), who includes a bibliography of the subject up to 1921.

A general knowledge of transmissive data in this region is of importance to all who use a radiometric detector with the filters noted above. Apparently many substances, if sufficiently thin, begin to transmit beyond 100 μ. Of the materials mentioned above for use as filters at the shorter wave lengths, those noted by Weniger which apparently have no transmission at these long wave lengths for one or more mm thickness are water and glass. Considerable transmission for one mm is found with amorphous quartz, fluorite, rocksalt, sylvite, and other substances. Crystalline quartz is notably transparent above 50 μ.

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APPENDIX

The above paper, in substance, was prepared during the winter 1924–25 for the International Critical Tables. Since its completion some

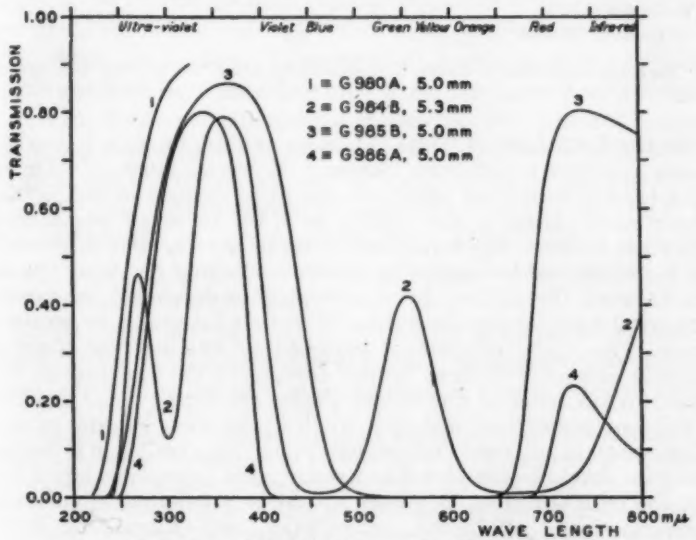


Fig. 4. — Special Glasses for Transmitting the Ultra-violet

remarkable glasses have been developed by the Corning Glass Works, these being notable for their very unusual transmission of the ultraviolet below 300 mμ, especially when the thicknesses of the glasses are considered.

In Fig. 4 the transmissions in the visible and ultraviolet are given. The data below 500 mμ are taken from B. S. Tests Nos. 44059 and 45120.[7]

The following points may be noted in connection with the use of these glasses as filters:

(1) The use of glass G 980 A in various thicknesses as a filter of class 1. There is practically no absorption between 300 and 800 mμ.

(2) The transmission of the ultraviolet below 300 mμ in the case of glasses G 986 A and G 985 B as compared to the similarly colored glasses illustrated by curves G_3 and G_4 of Fig. 3a.

(3) The approximate equivalence in transmission to curve D_2 of Fig. 3a, when G 985 B is combined with a red glass like G_{13} or G_{14} of Fig. 3a.

(4) In addition to the ultraviolet transmission of G 984 B, the shift in its transmission maximum in the green as compared with G_3 of Fig. 3a.

WASHINGTON, D. C.
JANUARY 22, 1926.

[7] The author is indebted to Messrs. H. J. McNicholas and F. K. Harris for these measurements.

On the Structure of Glass.—Glasses studied by x-rays have not shown any crystal structure (Scherrer, Wyckoff, Jauncey). On the other hand Lebedeff has attributed the rapid changes in the physical properties of glasses in the vicinity of 575°C to an allotropic transformation between two crystalline forms of quartz. But Schörnborn has found this critical region in glasses containing no silica. In the present work the authors have attempted to determine the inferior theoretical limit for the dimensions of crystals detectable by means of x-rays. The crystal structure of cristobalite β was determined experimentally and an hypothesis formed concerning the structure of cristobalite α. A series of crystallized glasses containing Na_2O and SiO_2 in different proportions was then studied, and these silicates of soda did not contain any form of crystalline SiO_2. The conclusion is reached that glass should be considered as a supercooled amorphous liquid, and that the critical regions in glasses should be attributed to molecular transformations. [Seljakoff, Stroutinsky, and Kiasnikoff; Revue d'Optique, *4*, pp. 545–554; 1925.]

G. W. MOFFITT

COLLIMATION OF LIGHT FROM A SOURCE OF FINITE EXTENSION*

By Ludwik Silberstein

In certain types of optical apparatus a possibly high degree of accuracy in collimating, i.e. rendering parallel, the rays of a beam of light forms an important requirement. In the abstract case of a point-source perfect collimation, with a perfect lens, is secured by placing the source at the focus of the lens. In practice, however, the relative dimensions of the light source not being negligible, the emergent rays will be more or less inclined to each other, even if the source, schematized as a flat surface, be placed in the focal plane of the lens. It has seemed interesting, therefore, to inquire whether there are perhaps other positions of such a source of finite extension which would give a nearer approach to perfect collimation and to actually determine the position of the source giving the best collimation. In order to investigate these questions a numerical measure of the deviation from perfect collimation must first be defined. As such a measure we will adopt, at Mr. L. A. Jones' suggestion, *the mean angle of inclination* of the emergent light rays to the axis of the lens.[1] This mean angle will be found by averaging first over the emergent rays due originally to any given point P of the source and then over the whole area of the source or, equivalently, of its image. To simplify the general expression of the required measure, the former averaging will be extended only to the rays contained in the meridian plane of the object point (P), such an approximate procedure being accurate enough for the applications of the formula sought for. This will amount to an integration over a single diameter of the diaphragm (aperture), to be followed by an integration over the whole area of the source or of its image.

Let the source or object be a disc of radius r placed with its center on, and normally to, the optical axis of a lens of focal length f and effective aperture $2R$. The lens will be schematized as a perfect optical system

* Communication No. 265 from the Research Laboratory of the Eastman Kodak Company. Paper read in abridgment at the joint meeting of the Optical Society of America with the American Physical Society at Montreal, February 26, 1926.

[1] We contemplate, of course, only beams to be collimated along the axis, not obliquely. Accordingly the (symmetrical) source is throughout assumed to be centered upon the axis of the optical system.

giving the familiar optical collineation between object and image. As a further simplification the two principal planes of this system will be assumed to coincide with each other. Let this common plane be the plane of the diaphragm HH (Fig. 1). Take the center O of the diaphragm as the origin of coordinates x, y and x', y' of an object-point P and of its image P', the abscissa x being reckoned positive in one and x' in the opposite direction along the optical axis, and y, y' being the

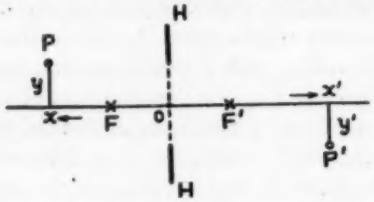


FIG. 1

distances from this axis. Then the well-known equations expressing the relation of image to object will become

$$(x'-f)(x-f) = f^2, \text{ or } \frac{1}{x'} + \frac{1}{x} = \frac{1}{f} \tag{1}$$

and

$$\frac{y'}{y} = \frac{f}{x-f}. \tag{2}$$

The focal length f will, under the said circumstances, be the distance of either focus, F, F', from the diaphragm. From (1) and (2) follows the simple relation

$$\frac{y'}{x'} = \frac{y}{x}. \tag{3}$$

Consider, in particular, a ray passing through the center O. If ω_0, ω_0' be its angles of inclination to the axis on entering and leaving the lens, the last equation gives

$$\tan\omega_0' = \frac{y'}{x'} = \frac{y}{x} = \tan\omega_0$$

or $\omega_0' = \omega_0$. Thus, under the simplifying assumptions any ray through the center O passes undeflected, or P' is collinear with O, P. The familiar construction of the image P' to any object P will therefore be reduced to drawing through P a parallel to the axis up to its intersec-

tion M with the diaphragm plane and to joining M with the focus F'; the intersection of MF' with the central ray PO will then be the image P' (Fig. 2). If P lies in the focal plane, the emergent pencil will simply

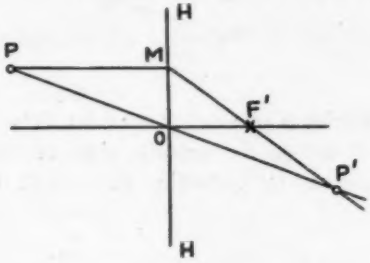


FIG. 2

consist of the parallels drawn through every point of the diaphragm to the central ray PO. In the general case all the emergent rays due to any object point P will be the straight lines joining the points of the diaphragm with the image P' just constructed, since in the ideal system substituted for the lens all rays from P pass through P'.

Turning to the problem in hand, consider first the special case in which the disc is placed in the focal plane, $x=f$. For any point $P(f, y)$ of the disc the inclination of the emergent central ray will, by (3), be given by $\tan \omega_0' = y/f$ or, since in all applications r/f and a fortiori y/f may be considered as small fractions,

$$\omega_0' = y/f .$$

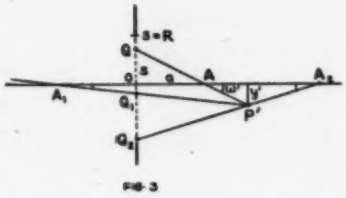
This will at the same time be the inclination ω' of the whole beam of emergent rays due to P, and therefore also the average over the diaphragm. It remains to average over the whole area of the disc. Thus the required mean inclination will become

$$\Omega = \frac{2\pi}{\pi r^2} \int_0^r \frac{y}{f}\, y dy = \frac{2}{3}\ \frac{r}{f} . \qquad (4)$$

Such then will be the measure of the deviation from perfect collimation for the focal position of the disc-shaped source.

Let now the disc be placed at any distance x from the diaphragm, restricted only by the condition $x \geqq f$.[2] Consider the rays from any point $P(x,y)$ of the disc. Let Q be any point of the diameter of the diaphragm

[2] The case $x<f$ to which corresponds a virtual image is without interest, the deviation Ω increasing steadily and rapidly when the disc moves from the focal plane towards the lens.



which lies in the meridian plane of P, and let $OQ=s$. Then, if Q and P', the image of P, are on the opposite sides of the optical axis, the inclination ω' of the ray QP', crossing the axis at A (Fig. 3), will be

$$\omega' \doteq \tan\omega' = \frac{y'}{x'-a} = \frac{s}{a},$$

where $a=OA$. Whence, eliminating a,

$$\omega' = \frac{y'+s}{x'} = \omega_0 + \frac{s}{x'},$$

where $\omega_0 = y/x = y'/x'$. This is to be averaged over the upper half of the diameter of the diaphragm, $s=0$ to $s=R$, giving

$$\overline{\omega'_1} = \omega_0 + \frac{1}{2}\,\frac{R}{x'}.$$

Next, take Q in the lower half of the diameter, i.e. on the same side of the axis with P'. Then, corresponding to the positions Q_1, Q_2 of Q (Fig. 3), respectively,

$$\omega' = \frac{y'-s}{x'} = \omega_0 - \frac{s}{x'}, \qquad \text{for } s \leqq y'$$

and

$$\omega' = \frac{s}{x'} - \omega_0, \qquad \text{for } s \geqq y'.$$

Thus, if $y' \leqq R$, the average $\overline{\omega'_2}$ over the lower half of the diameter of the diaphragm is

$$\overline{\omega'_2} = \frac{1}{R}\int_0^{y'}\left(\omega_0 - \frac{s}{x'}\right)ds + \frac{1}{R}\int_{y'}^{R}\left(\frac{s}{x'} - \omega_0\right)ds,$$

i. e.

$$\overline{\omega'_2} = \omega_0\left(\frac{y'}{R} - 1\right) + \frac{1}{2}\,\frac{R}{x'}.$$

The average over the whole diameter is $\bar{\omega}' = \frac{1}{2}(\bar{\omega}_1' + \bar{\omega}_2')$. Thus,

$$\overline{\omega'} = \frac{1}{2}\left(\omega_0 \frac{y'}{R} + \frac{R}{x'}\right), \qquad \text{for } y' \leqq R. \tag{5a}$$

If $y' \geqq R$, then throughout the lower half of the diameter

$$s \leqq R \leqq y', \qquad \omega' = \omega_0 - \frac{s}{x'},$$

so that

$$\overline{\omega_2'} = \omega_0 - \frac{1}{2}\frac{R}{x'},$$

and since for the upper half, as before,

$$\overline{\omega_1'} = \omega_0 + \frac{1}{2}\frac{R}{x'},$$

the average over the whole diameter becomes

$$\overline{\omega'} = \omega_0 = \frac{y}{x} = \frac{y'}{x'}, \qquad \text{for } y' \geqq R. \tag{5b}$$

The preceding equation rewritten in terms of the coordinates of the image point P' is

$$\overline{\omega'} = \frac{R}{2x'}\left(1 + \frac{y'^2}{R^2}\right), \qquad \text{for } y' \leqq R. \tag{5a}$$

The final mean inclination Ω can now be determined at once by averaging $\bar{\omega}'$ over the whole area of the image of the disc. Since this image is a circle of radius r', we have for $r' \leqq R$, by (5a),

$$\Omega = \frac{2\pi}{\pi r'^2}\,\frac{R}{2x'}\int_0^{r'}\left(1 + \frac{y'^2}{R^2}\right) y'\,dy'$$

and for $r' \geqq R$, by (5a) and (5b),

$$\Omega = \frac{2}{r'^2}\left[\frac{R}{2x'}\int_0^{R}\left(1 + \frac{y'^2}{R^2}\right) y'\,dy' + \frac{1}{x'}\int_R^{r'} y'^2 dy'\right].$$

Thus, evaluating the simple integrals,

$$\Omega = \frac{R}{2x'}\left(1 + \frac{r'^2}{2R^2}\right), \qquad r' \leqq R, \tag{6a}$$

$$\Omega = \frac{R^2}{3x'r'^2}\left(\frac{1}{4}+\frac{2r'^2}{R^2}\right), \qquad r' \geqq R, \quad (6b)$$

which is the required *measure of deviation from perfect collimation*, written in terms of the radius r' and the distance x' of the image of the luminous disc, and the constant R, the semi-aperture of the lens.

The variables x', r' may be expressed in terms of x alone, the distance of the disc from the diaphragm. In fact, by (1) and (2),

$$x' = \frac{fx}{x-f}, \qquad r' = \frac{rf}{x-f},$$

where r is the radius of the disc. Thus the last formulae become

$$\Omega = \frac{R(x-f)}{2fx}\left[1+\frac{1}{2}\,\frac{r^2}{R^2}\left(\frac{f}{x-f}\right)^2\right], \qquad x \geqq f\left(1+\frac{r}{R}\right), \quad (7a)$$

$$\Omega = \frac{1}{3}\,\frac{R^3(x-f)^3}{f^3r^2x}\left[\frac{1}{4}+2\frac{r^3}{R^3}\left(\frac{f}{x-f}\right)^3\right], \quad f \leqq x \leqq \left(1+\frac{r}{R}\right). \quad (7b)$$

As illustrative examples the following two particular cases may be considered.

First, if the disc is placed in the focal plane $x=f$, the formula (7*b*) or, equivalently, (6*b*) with $x'=\infty$, $r'=\infty$, has to be applied, giving

$$\Omega_\infty = \frac{2}{3}\,\frac{r'}{x'} = \frac{2}{3}\,\frac{r}{f}$$

which agrees with the previous result, (4). The suffix ∞ refers to the size of the image.

Second, if $r'=R$, that is to say, if the image of the disc is just as large as the aperture of the lens, both (6*a*) and (6*b*) give

$$\Omega_R = \frac{3}{4}\,\frac{R}{x'} = \frac{3}{4}\,\frac{r}{x},$$

and since this case arises for $x=f(1+r/R)$,

$$\Omega_R = \frac{3}{4}\,\frac{r}{f(1+r/R)}.$$

The ratio of these two values of the deviation is

$$\Omega_R : \Omega_\infty = \frac{9}{8} : \left(1+\frac{r}{R}\right).$$

Thus, for instance, if $r=5$ and $R=12.5$ mm, so that $r/R=2/5$, we have

$$\Omega_R : \Omega_\infty = 45 : 56 .$$

In this case, then, the $r'=R$ scheme would give a somewhat better approach to perfect collimation than the focal position of the source. For

$$f=100 , \qquad r=5 , \qquad R=12.5 ,$$

say,[3] both deviations are below two degrees, yet perceptibly different from each other, to wit

$$\Omega_\infty = 1/30 = 1°55', \text{ corresponding to } x=f,$$
$$\Omega_R = 1°32' \text{ corresponding to } x=1.4\,f.$$

For larger values of r/R the superiority of the $r'=R$ arrangement over the focal position of the source would be more marked.

Returning to the general formulae (6) or (7), the position of the source giving Ω minimum or the nearest approach to perfect collimation can be easily determined. For this purpose the form (7*a*), (7*b*) will be more convenient. In order to find the minimum of Ω and the corresponding position x of the light source, consider first the x-interval from $f(1+r/R)$ to infinity, in which formula (7*a*) holds. The derivative

$$\Omega'(x) = \frac{R}{2x^3}\left\{1 - \frac{1}{2}\,\frac{r^2 f}{R^2}\cdot\frac{2x-f}{(x-f)^2}\right\}$$

vanishes, apart from $x=\infty$, only for

$$\frac{x}{f} = \alpha\left[1 \pm \sqrt{1-1/\alpha}\right],$$

where $\alpha = 1+\frac{1}{2}r^2/R^2$. The discussion may henceforth be limited to the special numerical case $r/R=2/5$. For this case $\alpha=27/25$, and the greater of the two roots becomes $x/f=1.160$, while the interval in question is all beyond $x/f=1+r/R=1.40$. This root, therefore, and, the more so, the smaller root are outside the interval, and there is thus no minimum (or maximum) of Ω for $x \geqq f(1+r/R)$.

It remains to consider the interval $f \leqq x \leqq f(1+r/R)$, for which Ω is given by (7*b*). The derivative $\Omega'(x)$ vanishes only when

$$3x(x-f)^2 - (x-f)^3 = 8\left(\frac{rf}{R}\right)^3 \qquad (8)$$

[3] Ω retains, of course, its value if all linear dimensions are changed in the same ratio.

or

$$\left(\frac{x}{f}\right)^3-\frac{3}{2}\left(\frac{x}{f}\right)^2+\beta=0\,,\qquad \beta=\frac{1}{2}\left(1-8\frac{r^3}{R^3}\right),$$

which is a cubic for x. This is transformed by the substitution $x/f=u+\frac{1}{2}$ into the standard form

$$u^3+3pu+2q=0,$$

where $p=-\frac{1}{4}, q=\frac{1}{2}\beta-\frac{1}{8}$, i.e. again with $r/R=2/5, p=-\frac{1}{4}, q=-3/1000$. Since $q^2+p^3<0$, the three roots are real and may be written

$$u_1\,,u_2\,,u_3=2\sqrt{-p}\left(\cos\frac{\theta}{3}\,,\cos\frac{\theta+2\pi}{3}\,,\cos\frac{\theta+4\pi}{3}\right),$$

where $\cos\theta=-q(-p)^{-3/2}=24/1000$, $\theta=88°.625$. Thus $u_1=0.8700$, which falls within the limits of the interval in question, $0.5\leqq u\leqq 0.9$. The remaining roots u_2, u_3 being both negative, the only available solution is

$$\frac{x}{f}=\frac{1}{2}+0\,.8700=1\,.3700\,,$$

and since the second derivative, on substitution of (8), is

$$\Omega''(x)=\frac{R^3}{2r^2f^3}\,\frac{x-f}{x},$$

.e. positive, this position gives a genuine minimum. For $r=5$, $R=12.5$, i $=100$ the value of this minimum is, by (7*b*),

$$\Omega_{min}=0\,.02673=1°31'\,.9\qquad(x/f=1\,.37)\,.$$

This is perceptibly smaller than $\Omega=1°55'$ for the focal position of the source ($x=f$), but only very little smaller than

$$\Omega_R=0\,.02679=1°32'\,.\,1\text{ for }x/f=1\,.40\,,$$

the difference scarcely exceeding the limits of error due to the approximate treatment.

The optimal position of the light source can thus, in this case at least, be replaced by that which gives an image of the source equal in size to the aperture of the lens. These two positions fall near each other, being separated only by 0.03 f, and give practically the same approach to perfect collimation.

The discussion of other special cases corresponding to given numerical data can be easily dealt with by means of formulae (7*a*) (7*b*), and will be left to the reader.

It may still be noticed that beyond the optimal position the deviation Ω increases steadily with increasing distance, tending for $x=\infty$ to $\Omega=\frac{1}{2}R/f$ which in the case of the previous data is 3°35′ or more than twice the minimum value.

In what precedes the lens was substituted by an ideal optical system. The imperfections (aberrations, etc.) of the actual optical system will, of course, also contribute to the deviation Ω from perfect collimation. But these effects can scarcely be made the object of a general discussion and should rather be evaluated separately for optical systems specified for the particular nature of their imperfections.

ROCHESTER, N. Y.
MARCH, 1926.

On the Annealing of Glass.—One of the important factors to be taken account of in the annealing of glass is the considerable and rather sudden change in refractive index in a region determined by the critical temperature. The polymorphic transformations in the glass may cause changes in index amounting to several units in the third decimal place. The transformations take place in the same manner as those in solid solutions. Heat treatment of glass may result in lack of homogeneity in two ways, (1) presence of internal stresses, (2) non-homogeneity of structure, the effects of which may be much more pronounced than the effects of internal stresses. [A. A. Lebedeff; Revue d'Optique, *5*, pp. 1–30; 1926.]

G. W. MOFFITT

Electrodeless ring-discharge.—When a coil surrounding a closed tube is traversed by a radio-frequency current, and the density of the gas (air) within the tube gradually lowered, there appears first a luminous rose-colored ring discharge and then a brilliant white ring. This rather confusing paper apparently brings additional evidence (obtained by shielding the tube in divers ways) that the former discharge is due to the electrostatic field resulting from periodic accumulations of electric charge on the coil, and the latter to the magnetic field of the current.—[R. Wachsmuth and W. Schülz (Frankfurt); Ann. d. Phys., 78, pp. 57-71; 1925.]

KARL K. DARROW

NOTE ON ASPHERICAL LENS SYSTEMS

By Willard J. Fisher

In the paper by L. Silberstein, on aspherical lens systems, J.O.S.A., *11*, p. 479–494; 1925, the statement is made, p. 483, that to his knowledge the only previous method of closing a simple lens, whose front surface is generated by revolving a Cartesian oval, is by means of a spherical concave surface centered at the focal point of the ovaloid. In Eq. (7), p. 486, this case corresponds to making $f_1 = f_2$.

But if $\mu f_1 = f_2$, the second surface is again a hollow sphere, with one of its aplanatic points at F_1, the other at F_2, (Fig. 5, though without limitation to object at infinity).

This combination of surfaces is not aplanatic, by Eq. (15) and the paragraph following. The aplanatic condition being $h/\sin u = \text{const.}$, the deviation from aplanatism for the front surface is proportional to the departure from constancy of $1/(1-\cos u_1/\mu)$, in which $u_1 = 0_1F_1P_1$. If the back surface is the aplanatic spherical surface mentioned, and u_2 is the angle at F_2, then $\mu \sin u_2 = \sin u_1$.

The aplanatic condition can equally well be written $\sin u/h = \text{const.}$ Then the departure from constancy is proportional to $\cos u_1/\mu = \sqrt{1/\mu^2 - \sin^2 u_2}$.

For quartz glass and G light, $\lambda 4340$, $\mu = 1.467$. If $\cos u_1$ changes from $\cos u_1 = 1$, $u_1 = 0°$, to $\cos u_1 = 0.99$, $u_1 = 8°06'$, then $1 - \cos u_1/\mu$ changes from 0.320 to 0.327, or 2.2 per cent. Simultaneously u_2 changes from 0° to 5°30′. $2 \tan 5°30' = f/5.2$, roughly.

Such a quartz lens, computed for the transmission band of a silver film, might be of use in eclipse photography.

It may be mentioned that the writer has not seen any mention of the fact that a hollow sphere has aplanatic points. The above is a demonstration of the theorem, which was however discovered in an attempt to invert the Weierstrass construction, some years ago. As the theorem for a solid sphere is of great importance in microscopy, its inverse may equally become of technical importance.

DIFFRACTION GRATING IMAGES AND INTERFERENCE FIGURE IMAGES AS FORMED BY A MICROSCOPE OBJECTIVE

By L. V. Foster

A thesis written by Albert Driesen and published under the title of "The Influence of Some Aberrations of a Microscope Objective on the Image of a Diffraction Grating" appeared in the Zeitschrift fur Physik, October, 1923. Prior to this article O. Lummer and F. Reiche published Abbe's lecture on "The Study of the Image Formation in the Microscope," and K. Strehl published in Zeitschrift fur Instrumentenkunde, 1898, an article on the "Theory of the Microscope." Abbe's treatment of the image formation of non-self-luminous gratings in the microscope consisted of a study of the light distribution in the image plane (a plane conjugate to the object) and his investigations were confined to an aberration free objective. Strehl went further than this and studied the spatial light distribution in the region surrounding the image plane (the plane conjugate to the object) taking into account the influence of aberrations of the objective.

In 1910 H. Weisel investigated the light distribution in the neighborhood of a diffraction grating when illuminated perpendicularly by a parallel beam of light and found the region to contain what he calls stripe systems in planes parallel to the grating. Each system of stripes has the same spacing as the grating. These systems of stripes he found to lie at distances from the grating equal to nd^2/λ and each system of stripes is displaced in its own plane in a direction perpendicular to the length of the lines with respect to the system next to it by 1/2 the constant of the grating. The paper by Driesen treats mathematically and demonstrates by photographs the formation by a microscope objective of the images of these systems of stripes which are formed in close approximation to the grating. The author desired to verify the experiments which Driesen carried out and therefore took several photographs of a diffraction grating and the Wiesel system of stripes. It was found possible not only to reproduce the photographs obtained by Driesen but also to photograph the stripes under one set of conditions where Driesen failed.

The apparatus consisted of a microscope without body tube, having only a nosepiece for holding the objective and the usual coarse and fine

adjustment for focusing, a simple lens for substage condenser (numerical aperture less than that of the objective), and a pin-hole aperture located in its first principal focus, an arc lamp and a condenser to focus the arc image on the pin-hole, and light tight connections to an 8×10 camera. The grating used was a replica from a 14,000 line to the inch grating. Special supports were mounted in the front of the camera for holding the plates oblique to the optical axis of the objective and in a position such that the central portion of the plate passed near the geometrical plane conjugate to the object.

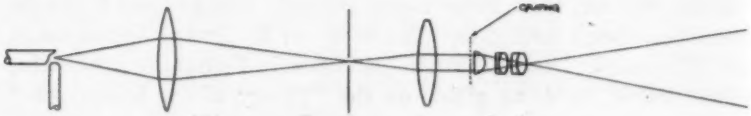


FIG. 1. *Apparatus.*

When the objective is focused on the grating, a small image of the pin-hole aperture is formed in the center of the second principal focal plane of the objective. Besides this there are two or more diffraction images of the pin-hole aperture located on either side of the center and in a line perpendicular to the lines of the grating. It is due, according to Abbe, to the presence of these diffraction images and the central image of the aperture that we see the lines of the grating clearly resolved. If an eyepiece is supported within the camera and the grating is brought into focus using a 4 mm objective the grating lines will be clearly visible in the center of the field but in concentric rings outside the center they seem to disappear and reappear. Upon focusing, with the fine adjustment, the lines in the center disappear and reappear over considerable depth of focusing. In just one particular focus the real image of the grating is seen, in any other position of apparent focus, what is really seen, is an image of one of the Weisel system of stripes, formed by the grating, which Driesen has called interference images. The concentric rings between zones of distinct focus of the grating lines are called boundary curves. These curves are intercepts of the focal plane of the eyepiece through the many image surfaces of the interference figures. The images which the microscope objective forms of the grating and its accompanying interference figures are not planes but are curved surfaces having their concave curvature towards the objective. These surfaces, the intercepts of which are seen in the microscope, are called boundary surfaces.

If, now, the eyepiece be removed and a photographic plate inserted in the microscope perpendicular to the axis and at the position of the image of the grating, a picture of the boundary curves can be taken.

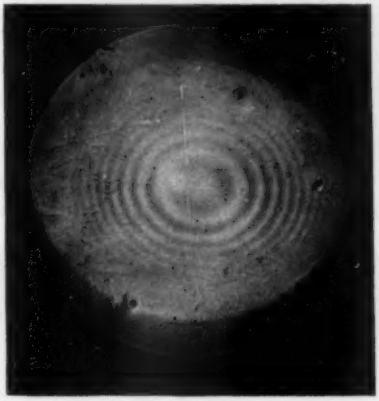


FIG. 2. *Boundary Curves. The plate is perpendicular to the axis of the microscope objective.*

(Fig. 2.) This picture and those to follow have been taken in this manner. If another plate is inserted in the microscope inclined to the axis about 10°, another intersection of the boundary surfaces will be photographed. The curves shown in this photograph represent the curvature

FIG. 3. *Intersection of the boundary surfaces and the photographic plate. The plate is parallel to the direction of the grating lines and inclined 10° to the axis of the microscope objective.*

of field of the microscope objective under consideration. If the plate be inserted so that its intersection with the plane of the grating is parallel to the lines of the grating, the photograph will contain images of the grating lines and images of their interference figures. The divisions between these images are the intersections of the boundary surfaces. Fig. 3 shows such a photograph taken with a 4 mm 0.85 objective. The plate can now be inserted so that its intersection with the plane of the grating is perpendicular to the lines of the grating and the photograph obtained will contain images of the intersections of the same boundary surfaces but at 90° from those in Fig. 3. Fig. 4 shows such a photograph taken with the same objective. Driesen shows pictures

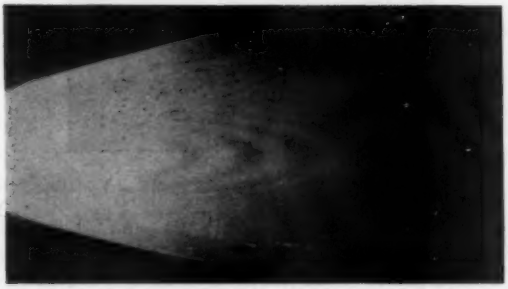


FIG. 4. *Intersection of the boundary surfaces and the photographic plate. The plate is perpendicular to the direction of the grating lines and inclined 10° to the axis of the microscope objective.*

taken with the plate parallel to the lines of the grating as shown in Fig. 3, but confesses not to have been able to demonstrate by photography the existence of curves taken through the other meridian, i.e., where the plate is perpendicular to the lines of the grating.

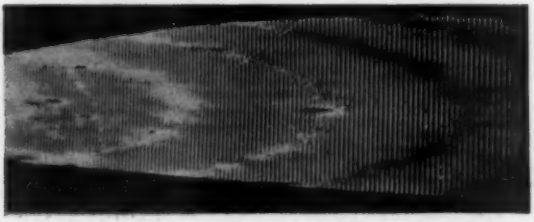


FIG. 5.* *Intersection of the boundary surfaces and the photographic plate. The plate is parallel to the direction of the grating lines and inclined to the axis of the microscope objective.*

* Fig. 5 is reproduced from Zs. f. Phys., October, 1923.

Figs. 5, 6, and 7 are photographs taken by Driesen with the plate parallel to the direction of the lines of the grating.

FIG. 6.* *Intersection of the boundary surfaces and the photographic plate. The plate is parallel to the direction of the grating lines and inclined to the axis of the microscope objective.*

FIG. 7.* *Intersection of the boundary surfaces and the photographic plate. The plate is parallel to the direction of the grating lines and inclined to the axis of the microscope objective.*

* Figs. 5, 6, and 7 are reproduced from Zs. f. Phys., October 1923.

FIG. 8.* *Boundary Curves. The plate is perpendicular to the axis of the microscope objective.*

* Figs. 6, 7 and 8 are reproduced from Zs. f. Phys., October, 1923.

Fig. 8 is a photograph taken by Driesen in which the plate is perpendicular to the axis of the microscope objective. This photograph shows the boundary curves.

Driesen points out that these curves are ellipses and show the astigmatism in the objective. This astigmatism is due to the formation of the image by at least one oblique pencil passing through the objective. Driesen has given a complete explanation of the ellipticity of these curves based on the fundamental theory of image formation by a microscope objective.

SCIENTIFIC BUREAU,
BAUSCH & LOMB OPTICAL COMPANY,
ROCHESTER, N. Y.

Helmholtz's Treatise on Physiological Optics. Translated from the Third German Edition. Edited by James P. C. Southall, Professor of Physics in Columbia University. Volume III, The Perceptions of Vision. Pages X+736, with 6 plates. Published by the Optical Society of America, Ithaca, N. Y. 1925. $7.00.

The first and second volumes of Helmholtz's Treatise on Physiological Optics, which have already been reviewed in this Journal in January and October, 1925, are considered to be primarily of interest to the physicist and the physiologist respectively. In the first volume the eye is presented as an optical instrument; in the second the sensations of color are reduced to an ordered system. The third volume, which is the subject of the present review, has for its purpose the minute study of the perceptions of vision, together with their philosophical significance, and is usually considered to be chiefly important to the psychologist.

The first volume contains 482 pages; the second 479; but the third comprises no less than 752 pages, which include a bibliography of nearly six hundred references to optical literature published from 1911 to 1925, indexes of subjects and authors covering the three volumes, two pages of corrigenda in Volumes I and II, and a few pages of plates. In addition to those of the German edition, numerous explanatory footnotes and special bibliographies of recent literature illustrative of the subjects discussed in the text have been inserted throughout the volume by the editor and his associates, and form most valuable contributions to the work. The reviewer, while reading the volume, has compared all the mathematical equations and the tables of numerical constants with the German text and noticed only a very few obvious misprints.

It might be supposed that with ten persons, whose names are recorded in the preface, co-operating with the editor in the work of translation of the volume, variations of style would be decidedly noticeable. The reviewer found an additional interest in reading the work in trying, but

without success, to distinguish the contributions of the different translators after the manner of the Biblical critics in their literary dissection of the Pentateuch. But so even has been the work of translation and so careful the editorial supervision that the present volume as well as the whole treatise possesses a uniform literary style.

In his prefatory note Professor Southall comments upon the difficulty of rendering the precise sense of the German text into English owing to the "metaphysical nature of many of the questions," which require the clear expression of the most delicate shades of meaning of the terms employed by Helmholtz himself and by von Kries also, the distinguished editor of this volume in the German edition, in his voluminous notes and discussions. Should Professor Southall and his accomplished collaborators entertain any misgivings regarding the precision of their translation, the reader will not question it, for the text possesses both vigor of style and clarity of expression in an eminent degree. To enable the discriminating reader to exercise his own judgment the editor has liberally introduced into the text pivotal German words and phrases upon which the ideas depend.

The scope of the volume is enormous. It includes such subjects as the laws of the rotations and other movements of the eyes, the monoc ular and binocular fields of vision, the perception of depth, binocular double vision, color mixing and contrast, Fechner's paradox, the physiological foundations of judgment and learning, and philosophical discourses on the rival theories of empiricism and intuitionism. At one extreme it discusses profound ideas regarding the concept of space, and at the other traces the optical illusions that terrify the unfortunate victim of delirium tremens to their origin in the irregular black spots that stand out against the intrinsic light of the retina. A suggestion of humor may also be suspected in the remark of Helmholtz concerning the imperfect co-ordination of the two eyes under conditions by no means unique: "when out of courtesy to the company I try to keep my eyes open after a long dinner, I am apt to see double images of the objects in front of me."

There is much in the volume of anatomical interest in the experiments and discussions on the movements of the eyes, especially their torsional rotations. The remarkably delicate and swift responsiveness of the muscles by which the movements of the two eyes are co-ordinated with precision, cannot be excelled or scarcely even equalled by anyother portions of the anatomical structure. The detection and measurement of the torsional rotations have required the development of a multitude of the most exact and ingenious experiments in which Helmholtz himself was most fertile. So naturally and unobtrusively are these introduced into the text that the reader may easily fail to realize the magnitude of the contribution of Helmholtz himself to this part of the subject. Most remarkable of all the motions of the eyes, however, appears to be the peculiar movements described by von Kries (p. 128) when he concludes that under certain conditions "the motions are of the general

form of a screw motion, that is, they consist of a combination of a rotation around an axis together with a displacement parallel to the axis."

To the movements of the eyes, which obey the laws of Donders (p. 44), and Listing (p. 48), a very large amount of consideration is given, including about thirty-five pages of mathematical investigation. In general a movement of the eyes from one point of fixation to another involves a torsional rotation, except that one position "can be found such that when the eyes turn from it to look straight up or straight down, or straight to the right or left, there will be no rolling of the eye" (p. 44). Helmholtz (p. 145) showed that these rolling movements of the eye obeyed the principle of easiest or least orientation, that is the movements are such that the least effort is required to effect them. This involves the least effort in training the eye to perform useful and easy movements, and eliminates the necessity of training it to perform those that are useless or difficult.

There appears to be a law of economy omnipresent in nature. In dynamics it is known generally as the principle of least action, in light, of least time, in the distribution of electric currents in a net work of conductors as the principle of least heat. In muscular actions Sherrington has shown that the opposing muscles are reciprocally innervated, by means of which movements are most rapidly and economically effected. The muscles of the eyes are subject to this principle, and therefore Helmholtz's law of least orientation is but one example of Sherrington's general principle. The subject of reciprocal innervation has been developed since the time of Helmholtz; but it does not appear to have occurred to von Kries to make use of it in his discussion of Helmholtz's law. It seems quite possible, if not indeed probable, that from Sherrington's principle the laws of Donders, Listing and Helmholtz can all be derived as necessary consequences of its operation, just as Kepler's laws of planetary orbits were deduced by Newton from his law of gravitation.

It is obviously of the greatest advantage to have in the same volume the conclusions of so profound an investigator as Helmholtz, and in addition summaries of the extensive researches on the same subjects that have accumulated since his time. These have been brought together by von Kries and elaborately discussed in relation to the questions considered by Helmholtz. The views of Helmholtz were not lightly adopted by him, but represent the considered conclusions after weighing the evidence available from every point of view. In many, probably most, cases these opinions are confirmed; in others they are disproved or modified. It would indeed be extraordinary if in so vast a subject as vision the judgments even of a Helmholtz should invariably be correct. We are not surprised therefore to find divergences between the conclusions of Helmholtz and those of other great investigators in the same field.

In the highly interesting question, for example, of determining the direction of vision, Helmholtz lays great stress (p. 243) on the function

of "muscular feeling," which, he claimed, "makes itself felt in the muscle by a contraction which actually takes place." The reasoning seems satisfactory and convincing enough; but on turning to the Appendix by von Kries, (p. 604), we learn that it is extremely doubtful whether the "feeling of innervation" exists. "At any rate," says von Kries, "it cannot be brought clearly to consciousness in any satisfactory way". From the argument of the latter we are further assured that in so far as the determination of the "impression of locality" is concerned, "it does not make much difference whether these innervations are accompanied or not by any feeling of which we can be conscious."

The origin of giddiness is another of the phenomena in which the explanation of Helmholtz (p. 249) is superseded by later investigation (p. 278).

Only a very small part of the volume is concerned with color vision. The color theorist, however, will be interested in the discussions of binocular color mixing, binocular contrast and Fechner's paradox. Helmholtz (p. 505) is most positive that he cannot observe any undoubted binocular mixing of colors, and while admitting the observations of others, yet he suggests explanations of them in conformity with his own views. On this matter von Kries (p. 528) cites experiments which show clearly that the phenomena in question do exist and describes the method by which they can readily be seen. On this point of much theoretical importance Helmholtz seems clearly to have been wrong.

Quite as important from the standpoint of color theory are the subjects of binocular color contrast and Fechner's paradox. In considering the latter, whereby stimulation of one retina by dim white light lowers the brightness of light perceived by the other, Helmholtz falls back upon his explanation of simultaneous contrast. "One interpretation that might be given to this paradoxical experiment," he observes, "is to suppose that, under certain conditions, the sensation of light in one eye tended to lower that in the other eye, as if there were some antagonism between the two retinas. However by making a slight modification in the experiment, I have been able to show that something of an entirely different nature is involved here." After describing experiments of his own in which the surfaces viewed are very large, he says (p. 523): "This variation of Fechner's experiment shows that there is no question here of a change in the sensation of the light, but that it is simply a matter of changing our opinion as to the real color of the white object."

Helmholtz's experiment, however, is different from Fechner's, since the stimulation of the periphery of the retina is altered in character. Therefore it does not in the least degree explain Fechner's. The real explanation lies not in the deception of the judgment, but in the reciprocal influence of one retina upon the other through sensory reflex action, which, in the case of dim light, depresses the sensitivity of the visual receptors.

The volume opens and, as far as its major interest is concerned, closes with philosophical discussions of the rival theories of empiricism and intuitionism or nativism, particularly in regard to the origin of the concept of space.

As Helmholtz had adopted the empirical theory and had urged it with all the force of argument and wealth of experimental evidence at his command, it is important to discover how far his attitude is justified by the lapse of time. The present state of the controversy is developed at great length by von Kries who shows clearly both the strength and the weakness of Helmholtz's position. In the hands of Helmholtz empiricism completely excluded intuitionism, on which account he was brought into conflict with the general trend of philosophical thought in Germany, and especially with the views of Kant. This phase of the situation is analysed with great care and skill by von Kries, who shows that in developing his theory of localization, which subsequent consideration has failed to shake or even seriously to modify, Helmholtz needlessly ruled out the intuitional views on the profounder question of the subjectivity of the idea of space. Thus von Kries says (p. 638): "For anyone who takes our attitude toward the subject, it must be a source of regret that although in the main, we can accept Helmholtz's empirical theory of localization, which after all is the matter in which we are chiefly interested here, he laboured under a misapprehension especially in regard to Kant's doctrine of the apriority of spatial intuition, and also in regard to the relation of the various problems to one another. The result was that he was placed in a position of antagonism to Kant, although such a position is not necessary for an empirical theory of localization. It also prevented him from grasping fully the distinction between space-determinations and qualities of the senses." And again on the same page von Kries says: "However, a further complication resulted from the fact that Helmholtz believed that not only the special relations of localization, but the idea of space itself had to be regarded as acquired by experience. Not to mention the difficulties in which this involved him in other parts of philosophy, so far from helping to clarify the theory of localization, it tended to obscure it to a very great extent, and indeed deprived it of an indispensable basis. . . . . The truth is that by trying to extend empiricism to the apperception of space as such, Helmholtz constructed a theory which had an hiatus in it and was left hanging in the air at the place where it was needed to have a solid support."

It will be no easy task to combat the powerful arguments of philosophy, especially of the Kantians, regarding the intuitional basis of the form of space. The extremes of empiricism and intuitionism, therefore, must evidently become merged into some kind of Hegelian synthesis unless the controversy between them is to be perpetually maintained.

It is extremely doubtful, however, whether the scientific or the philosophical world will accept the general and very materialistic conclusion

of von Kries, that applies equally though in different ways to the doctrines, or perhaps it would be more correct to say the dogmas, of Helmholtz and Hering also, which is thus summarized (p. 642): "it is now pretty generally admitted in scientific circles at least (in spite of the fact that there is considerable dispute about it), that the substrata of all the phenomena of consciousness even in their most minute details are to be found in the physical processes of the brain."

From the standpoint of scientific research nevertheless it will be conceded that empiricism must always enjoy an enormous advantage over its rival, for reasons which von Kries urges in concluding his lengthy discussion (p. 651): "From this point of view the position taken throughout this discussion might be said to be a fundamentally empirical conception, in spite of the fact that it does contemplate the possibility of innate bases. Anyone who will follow the argument as here presented will see that the principle which has guided us, and which remains still today the best way of obtaining an insight into these problems and is the basis of future investigation, has been the empiricism of Helmholtz, even though it has had to be modified and amplified in many respects."

It has been stated, as remarked above, that the first volume of the Physiological Optics is for the physicist, the second for the physiologist, and the third for the psychologist. This is true in the sense that the problems considered in each volume largely belong to their branches of science, and may accordingly be consulted by investigators in those domains. To imply, however, that the volumes are but sectional in their interest is to fail completely to grasp the scope and purpose of Helmholtz's work. Indeed, it is only by reading the third volume that one clearly grasps the comprehensiveness, as well as the successful elucidation, of the idea underlying the whole, which renders the work a scientific classic of the first magnitude. Beginning with the description of the organs of vision Helmholtz employs the resources of anatomy to explain accommodation, of physics and mathematics to investigate the formation of the retinal image, of physiology to analyse the nature and relationship of the colors perceived, of psychology and philosophy to account for the perception of depth, the apperception of space and the orientation of the individual within it. The evidence from so many diverse sources is marshalled with consummate skill to solve perhaps the profoundest problem of the natural world with which the mind is confronted.

The reviewer must confess to experiencing at first a considerable degree of tedium when beginning to read the third volume. But as its purpose was discerned the interest became aroused and was sustained not only through the portions due to Helmholtz but throughout the lengthy discussions of von Kries as well. No one who has read the first two volumes should fail to read the last even though initially it might appear somewhat dry or even repellant. Probably a careful perusal of Atkinson's translation of Helmholtz's popular address; "The recent (1868) progress of the theory of vision," would serve as an excellent

introduction to the Physiological Optics of which it is a brilliant summary.

The preparation and publication of a work of such importance and magnitude obviously involve questions of the most diverse kinds. The business and financial considerations alone must have required competent judgment and the most generous gift of time from the inception to the completion of the undertaking. The Optical Society of America, which conceived the idea of translating the work, and the editor, Professor Southall, enjoyed the good fortune of having the co-operation of Mr. Adolph Lomb. The nature of the services rendered by this gentleman may partly be inferred from his well known business connections. But the references in the editor's introductory notes in the first and third volumes, indicate that Mr. Lomb's interest far exceeded the business problems inseparable from a work of this character. His unfailing enthusiasm, his eagerness to promote the advancement of optical science, and his participation in the translation of the third volume, will merit and receive the gratitude of students of visual science for many generations. The reviewer was inclined to pay this tribute when the first volume was under consideration, but decided to refrain until he had read the complete work. Now that he has done so, he can more vividly realize the magnitude of Mr. Lomb's contribution to the success of the undertaking.

It is unlikely that the Optics will ever be superseded or perhaps even equalled. Men of the ability of Helmholtz will no doubt arise from time to time. But it seems improbable that such minds will be devoted to the group of sciences necessary for the purpose, unless in process of time investigation should bring to light such extensive and radically different types of phenomena that the present treatment will be rendered wholly inadequate. Professor Southall may rest content in the certainty that until that event happens his translation will remain a great English as the original will a great German classic of science.

FRANK ALLEN

The Laws of Pupillary Constriction.—The author shows especially how the pupil varies in an unaccommodated eye in binocular vision, and then in monocular vision, as a function of the illumination with white light of an unlimited field of adaptation. The curves of constriction are similar to those of visual acuity. The influence of the apparent diameter of the field and of its position with respect to the axis, as well as that of the state of accommodation of the eye, are then studied. Finally, consideration is given to the manner of the pupillary contraction, its duration and the modifications it causes in all the phenomena of fatigue of the eye. [Jacques Couvreux; Revue d'Optique, *4*, pp. 555–561; 1925.]

G. W. MOFFITT

Mid Winter Meeting of The Optical Society of America

Held jointly with the American Physical Society at McGill University, February 26 and 27, 1926.

A special meeting of the Optical Society of America was, by invitation, held jointly with the American Physical Society at McGill University, February 26 and 27, 1926. In addition to the contributed papers, abstracts of which appear below, the following special features of the meeting deserve comment.

Under the auspices of the Optical Society of America, Professor Frank Allen, Professor of Physics of the University of Manitoba, by invitation, gave a lecture on Friday afternoon at three o'clock, on the subject, "The Second Half of Vision".

On Friday evening, members of the visiting societies were the guests at dinner of the Chancellor, the Principal, and the Board of Governors of McGill University.

The Optical Society is much indebted to McGill University for its hospitality.

The following papers were presented. (For abstracts of papers presented before the American Physical Society, see Minutes of this meeting as published in the PHYSICAL REVIEW.)

F. K. Richtmyer,
Sec. Pro tem.

1. Ludwik Silberstein — **Research Laboratory, Eastman Kodak Co.**

Collimation of light from a source of finite extension.

For a point-source perfect collimation, with a perfect lens, is obtained by placing the source at the focus of the lens. If, however, the dimensions of the source are not negligible, the emergent rays are inclined to each other and to the optical axis, even if the (flat) source be placed in the focal plane. It then becomes of interest to inquire whether some other position of the light source gives a nearer approach and to actually find the position giving the nearest approach to perfect collimation. This is the object of the paper.

As a numerical measure (Ω) of the deviation from perfect collimation is defined the mean value of the angle of inclination of the emergent rays to the axis of the optical system, this mean being obtained by averaging first over the rays issuing from a fixed point of the source and then over the whole area of the source or, equivalently, over that of its image. A formula for the deviation Ω thus defined is constructed for the case of a disc-shaped source and is discussed in general terms and on a numerical example illustrating the dependence of Ω upon the distance and the size of the source and upon the effective aperture of the lens. The position of nearest approach to perfect collimation, which turns out to be extra-focal, is determined and the corresponding minimum deviation Ω is compared numerically with that belonging to the focal and to some other positions of the source.

The present paper will appear in full in J. O. S. A. and R. S. I.

15 min.

(Received January 19, 1926)

2. Dr. Clayton H. Sharp — **Electrical Testing Laboratories**

Various applications of the Photoelectric Cell with Amplifier to Photometry.

A.—An arrangement whereby a photoelectric cell with a vacuum tube amplifier supplants the ordinary photometer head in making a photometric balance has been

described before the Illuminating Engineering Society in September 1925. A rotating glass disc, one half of which is silvered, throws in alternation the light of the "X" lamp and the light of the comparison lamp on to the photoelectric cell, the cathode surface of which is connected to the grid of an amplifying tube. If the two lamps differ in their effect on the cell, variations in the plate current are produced which act through a transformer and through a rectifying device upon a DC galvanometer. Equality is indicated by zero deflection. Experience has shown that unusual accuracy and speed can be obtained in photometric work with this device which is in daily operation measuring more than 400 lamps per day.

B.—Spectrophotometry.

The filament of an incandescent lamp is focussed on a spectrometer slit, the beam passing through a variable sector. The photoelectric cell with amplifier is the detector. A circuit connection is used such that the galvanometer shows only the changes in the plate current. Spectrophotometric measurements of transmission are made by noting the deflection of the galvanometer when the filter is interposed in the beam, and then by means of the sector bringing the galvanometer to the same deflection when the filter has been removed.

C.—Temperature of Lamps.

The ratio of light transmitted by a blue filter to that transmitted by an amber filter as shown by the photoelectric cell varies with the temperature of the source of light. The temperature of incandescent bodies can thereby be determined, using a calibration curve.

In using the photoelectric cell with an amplifier, expecially when the amplifier has no grid leak, very thorough electrostatic shielding is essential Low frequency magnetic distrubances are the most difficult to eliminate. 10 min.

(Received January 21, 1926)

3. L. A. Jones and V. C. Hall **Research Laboratory, Eastman Kodak Co.**

ON THE RELATION BETWEEN TIME AND EXPOSURE IN PHOTOGRAPHIC EXPOSURE.

Further results of the study of the reciprocity relation between the time of exposure and the intensity of exposing radiation are given. It has been found that the maximum density obtainable with complete development is dependent upon the intensity used in making the exposure. This indicates that for the silver halide grains there is an intensity threshold below which developability cannot be produced, no matter how long the exposure time is prolonged. It has also been found that if the exposing intensity be sufficiently high all of the silver halide present in the emulsion is made developable.

The present paper will appear in full in J. O. S. A. and R. S. I. 20 min.

(Received January 20, 1926)

4. Harley E. Howe **Cornell University**

THE COLOR TEMPERATURE OF THE GAS FILLED TUNGSTEN LAMP AS A FUNCTION OF TIME IN SERVICE.

Certain gas-filled lamps, used as secondary standards of color temperature, were found to require an increase in voltage and current as they aged in order to maintain them at 2848°K. To test for uniformity of behavior, groups of lamps of four different types were put on a life test at voltages that initially brought the lamps in each group to 2848°K. At intervals the color temperature of the lamps was measured with the rotatory dispersion colorimeter at the Bureau of Standards.

Each lamp showed erratic variations from a mean temperature, the average range covered being about 60°. For a lamp not under life test, a large number of observations

distributed in time gave temperatures covering a range of 30°. The lamps tested at 2848°K showed no indication of a definite aging effect.

Three 9 volt headlight lamps, burned at the voltage necessary to bring them initially to 3000°K, showed drops of 100° during their life of 40 hours.

The effect of preliminary "seasoning," such as is carried out with photometric standards, was less than the observational error in temperature measurements.

The author concludes that in color temperature measurements frequent checks against primary standards should be made. For rough work only, constancy of color temperature at a constant voltage may be assumed. 10 min.

(Received January 21, 1926)

5. **K. S. Gibson** **Bureau of Standards**

THE PRODUCTION OF RADIANT ENERGY OF UNIFORM INTENSITY OVER THE VISIBLE SPECTRUM.

It has been found desirable in certain colorimetric work at the Bureau of Standards to have a filter which would reduce the relative energy of the Abbot-Priest artificial average noon sunlight to a constant energy value throughout the visible spectrum. The attempt was made to reproduce the ideal filter for this purpose as closely as possible spectrally, with the unavoidable variations of such a nature that the dominant wave-lengths and purities of the actual and ideal filters should agree within the respective limens.

Considerations of permanence and convenience in handling favored the use of colored glasses in preference to solutions or dyed gelatine films. A study of the available glasses resulted in a choice of the following two in combination:

1. Am. Opt. Co. Amethyst C, thickness 1.26 mm. (Cf. B. S. Tech. Pap. 119, Fig. 15 curve C.)

2. Ives-Brady Daylight Glass, thickness 0.30 mm.

The following data will be illustrated:

1. The relative spectral energy of Abbot-Priest artificial average noon sunlight.

2. The spectral transmission of the ideal filter required to reduce this sunlight to equal energy.

3. The spectral transmission of the actual filter (above specified) designed to duplicate the ideal filter.

4. The relative energy of Abbot-Priest sunlight after transmission by the actual filter.

The agreement in color between the actual and ideal filters may be noted from the following table:

Dominant Wave-Length and Purity of Filter, Referred to Average Noon Sunlight (Abbot-Priest) as Neutral Standard.

1. Computed on basis of O. S. A. "Excitations" and "Luminosity Coefficients" (from spectral transmission)	Dominant Wave-length	Purity
Ideal filter	569.0C	—0.22
Actual filter	568.2C	—0.21
2. Observed (actual filter) K. S. G.	568.6C	—0.18

Bibliography;
Priest: Phys. Rev., (2), *11*, p. 502. Cf. "Standard Artificial Sunlight": 10th Annual Meeting, O. S. A.
5 min.

(Received January 20, 1926)

6. Irwin G. Priest **Bureau of Standards and Munsell Research Laboratory**

AN EXPERIMENT BEARING ON THE ADOPTION OF A STANDARD NEUTRAL STIMULUS IN COLORIMETRY: THE CHOICE AS BETWEEN "SUN" AND "EQUAL ENERGY."

Twenty-five observers viewed a photometric field under the following conditions: (1) *Shape*: Square, divided vertically into two halves. (2) *Size*: Side of square subtends 4°. (3) *Brightness*: 2 to 4 millilamberts. (Two halves equal, and surrounding field dark.) (4) *Pupil*: Circle, 3 mm diam.

One-half was illuminated by standard artificial noon sunlight, the spectral distribution in the other half being a close approximation to equal energy for all wave-lengths in the visible.

The names assigned to the colors of the halves of the field were, in general, somewhat as follows:

Sun: Yellow, green or greenish yellow.

Equal Energy: Pink, purple or blue.

When either stimulus acts alone (with dark surrounding field), the color is readily admitted to be neutral in either case; but when in juxtaposition, *neither* appears neutral, one or two observers being excepted. In general, for the most observers, both colors, even in juxtaposition, are very pale tints. One observer, on her own initiative, called the sunlit half "greenish yellow" and the other "white". Others had to be urged to choose which color was the closer approximation to white, i. e., when seen together in juxtaposition which was the *less saturated*. With two or three exceptions, the observers were in complete ignorance of the nature of the experiment.

The vote for less saturation (or closer approximation to "white") stands as follows:

For equal energy without doubt	15
For sun without doubt	3
For equal energy, but doubtful	1
For sun, but doubtful	2
Unable to see difference	2
Able to see difference but unable to decide	2
Total	25

This result suggests that an "equal energy source" is worthy of serious consideration as a neutral standard; but, pending other experiments, we refrain from definitely recommending it.

Bibliography:
Priest: B. S. Sci. Pap. 417 or J. O. S. A. *5*, pp. 205–209, 1021.
Priest: "Standard Artificial Sunlight," 10th Annual Meeting, O. S. A.
Gibson: "Equal Energy Spectrum," this program.
Helmholtz: Physiol. Op. O. S. A. Eng. Trans. vol. 2, p. 274.

10 min.

(Received January 20, 1926)

7. Irwin G. Priest and F. G. Brickwedde **Bureau of Standards and Munsell Research Laboratory**

THE MINIMUM PERCEPTIBLE COLORIMETRIC PURITY AS A FUNCTION OF DOMINANT WAVE-LENGTH WITH SUNLIGHT AS NEUTRAL STANDARD.

Method and Conditions: A 4° square, two-part, field, symmetrical about a vertical division and viewed through a pupil 3 mm in diameter, is illuminated in both parts by artificial sunlight, at a constant brightness of about 3 or 4 millilamberts, with a surrounding field of about 0.5 millilambert. Homogeneous light is added to one-half, and sunlight simultaneously subtracted so the field remains matched in *brightness*. We attempt to make two adjustments of the mixture: (1) The least purity perceptible with certainty, (ρ_{max}); (2) The greatest imperceptible purity, (ρ_{min}). Care being taken to correct for stray light, the

purity of these mixtures is then measured, accuracy being obtained by measuring a *known large multiple* of the homogeneous brightness.

Results: (1) For shortest wave-lengths, the mean of ρ_{max} and ρ_{min} is very small (0.001 or less); (2) increases slowly to about 0.02 at 550 mμ; (3) rises rapidly to a sharp maximum at about 570 and falls still more rapidly to about 0.02 at 580; (4) there is then a gradual fall to less than 0.01 at 640 mμ beyond which we have no data. For Brickwedde the wave-length of the maximum (precisely determined) is 568 mμ, the values for ρ_{max} and ρ_{min} being respectively 0.062 and 0.024. The corresponding wave-length for Priest (less precisely determined) is probably greater but not more than 572 mμ.

Remarks: 1. Least perceptible purity is largely dependent upon visibility; the relative amounts of *energy* required to introduce a hue difference show comparatively small variation with wave-length.

2. The wave-length at which least perceptible purity with sunlight as neutral is a maximum approximates very closely to the dominant wave-length of sunlight referred to equal energy spectrum as neutral. On the basis of the O. S. A. Colorimetry Committee "excitations" the latter wave-length is 569.0 mμ.

3. Further extensive investigation of least perceptible purity should be prefaced by a definite adoption of a neutral standard.

Bibliography:
(1) Priest: J. O. S. A. & R. S. I., 8, pp. 173-200 (for definition of purity and description of apparatus).
(2) Priest: "Standard Artificial Sunlight," 10th Ann. Meeting, O. S. A.

(Received January 20, 1926) 15 min.

8. W. F. Meggers and C. C. Kiess **Bureau of Standards**

SPECTRAL STRUCTURES FOR ELEMENTS OF THE SECOND LONG PERIOD.

In 1924 a summary of the regularities found in arc and spark spectra of elements in the first long period (K-19 to Fe-26) was published[1] and the *displacement law* of spectroscopy was verified across the entire chemical table. During the past year similar results have been obtained for some elements of the second long period (Rb-37 to Pd-46), and the main features of these spectra are now presented because of their interest for the development of the theory of multiplet structures. The spectra dealt with are those of Rb, Sr, Y, Zr, Cb, Mo, ekaMn, Ru, Rh, and Pd, most of the details being for the four elements following Sr. Typical multiplets representing systems of various permanent multiplicities and involving the most important spectral terms are given for each spectrum. The *displacement* and *alternation* laws are again found to be strictly valid, but some important differences between the spectra of the elements in the second long period and those of analogous elements in the first long period are noted. In connection with the empirical data, it is pointed out that the rule governing interval ratios of complex spectral terms is more frequently violated in spectra of heavy elements and deviations from the Landé g factors defining the Zeeman patterns of the spectral lines are not uncommon. The most sensitive lines for the purpose of spectrographically detecting a chemical element (*raies ultimes*) which were formerly believed always to involve the spectral term of lowest energy (normal state of the atom) appear in a few cases (Mo II?, Pd I) to originate with somewhat higher energized states when the corresponding terms are represented by considerably larger quantum numbers.

The normal states of the neutral atoms Rb to Pd are represented by the spectral terms

$$^2S, {}^1S, {}^2D, {}^3F, {}^4D, {}^7S, {}^6S?, {}^5F, {}^4F, {}^1S,$$

and the normal states of the ionized atoms Sr to Mo by the terms

$$^2S, {}^3D, {}^4F, {}^5D, {}^6S?,$$

respectively.

Bibliography:
1. Meggers, Kiess and Walters, Jr., J. O. S. A. & R. S. I., 9, 355; 1924.

The present paper will appear in full in J. O. S. A. and R. S. I. 15 min.

(Received January 20, 1926)

9. Irwin G. Priest **Bureau of Standards and Munsell Research Laboratory**

BLUE SKY AND WHITE SNOW

A Note on Sensation and Perception.

(Illustrated by an Illusion Discovered by Prof. C. A. Skinner.)

This paper is the sequal to a paper, "Gray Skies and White Snow," (Physical Society, Optical Society, N. Y., February, 1925). A beautiful illusion, discovered by Skinner will be described and used as an introduction to the discussion of the perception of saturation. In the previous paper it was shown that the brightness of an area of overcast sky perceived as "gray" might be greater than that of an area of snow perceived as "white," and that the "value" of a perceived color depended upon the observer's opinion as to whether the area in question was sky or snow. In this paper it is shown that similar relations exist in regard to saturation.

On a clear sunlit day with blue sky, cast shadows on the snow are blue. Physicists and other clear thinkers must admit that this is true ("objectively"); artists and other keen observers see it so. However, it appears that the highest degree of saturation of the blue shadows is not experienced while the observer is fully conscious of the snow. Let this consciousness be eliminated (or merely reduced in intensity) and the saturation increases in a most marked degree. Distant snow banks assume the illusory character of "equivocal figures," being perceived in alternate moments now as white snow banks in shadow and now as *blue lakes*, without any change in objective conditions (Skinner's illusion).

In the image on a camera ground glass, the shadows on snow are much more saturated than in a direct view of the landscape.

It seems certain that no physical hypothesis as to conditions external to the observer can account for these "illusions." It does not seem likely that they can be explained by retinal conditions. They are rather in accord with Helmholtz's ideas of perception.

The fundamental principles involved in this and the previous paper are not new; but they have not received the general recognition which they merit. They have a direct and important bearing on the problems of colorimetric standards and nomenclature, indicating that the nomenclature and standards of colors in *perception* is a much more complex subject than in mere sensation and deserves in itself more careful consideration than it has been accorded.

Bibliography:

1. Priest: J. O. S. A. & R. S. I, *11*, p. 133; August, 1925.
2. A. H. Munsell: "A Color Notation" or Century Dictionary (for definition of "value").
3. "Report of Colorimetry Committee" J. O. S. A. & R. S. I., *6*, p. 527.
4. Helmholtz: Physiol. Op. O. S. A. Eng. Trans. Vol. 2, pp. 131, 272, 297, (footnote 17), and 300; and Vol. 3, p. 286.
5. Luckiesh: "Visual Illusions" (N. Y. 1922), Chap. V.
6. Troland: "Mystery of Mind" (1926), pp. 88–96.

10 min.

(Received January 20, 1926)

FINANCIAL REPORT

Journal of the Optical Society of America and Review of Scientific Instruments For the Period from January 1, 1925 to December 31, 1925

RECEIPTS

Cash on Hand, January 1, 1925		$ 2,281.62
To apply on Contract of the Optical Society		
Subscriptions	$ 48.88	
Sale ov Back Numbers of Volumes I-V	224.90	
Advertising collected for Optical Society	572.33	
Payments from Treasurer of Optical Society	3,940.64	
Sale of Reports of Committee on Colorimetry	18.55	
Total		4,805.30
To apply on Contract of the Association of Scientific Apparatus Makers of the United States of America		
Advertising		8,563.72
Miscellaneous Receipts		
Subscriptions	$ 2,752.77	
Sale of Back Numbers of Volumes VI-XI	291.97	
On account of Helmholtz Translation (contra)	21.00	
On account of Membership Fees (contra)	15.00	
Reimbursement for expenses of Optical Society Meetings (1925)	89.56	
(1924)	81.07	
Sundry items	81.62	
Total		3,332.99
Grand Total		$18,983.63

DISBURSEMENTS

Printer	$10,586.57	
Salaries	4,331.95	
Supplies	323.31	
Miscellaneous (Postage, etc.)	528.15	$15,769.98
Cash on Hand, December 31, 1925		3,213.65
		$18,983.63

The above is a correct statement of accounts to the best of my knowledge and belief.

(Signed) F. K. RICHTMYER, Business Manager.

We, the undersigned, duly appointed to audit the accounts of the Journal of the Optical Society of America and Review of Scientific Instruments as at the close of business, December 31, 1925, hereby certify that we have examined the statement of receipts and disbursements, the vouchers, check book and bank book and find them to agree with each other. We, there-

fore, certify that the above statement is a true statement of the accounts of the Journal of the Optical Society of America and Review of Scientific Instruments as of December 31, 1925.

January 20, 1926.

(Signed) WALTER H. STAINTON
LEO BEHR
Auditors.

OPTICAL SOCIETY OF AMERICA

STATEMENT OF CASH RECEIPTS AND DISBURSEMENTS

For the Period from January 1, 1925 to December 31, 1925

CASH ON HAND, JANUARY 1, 1925		$ 8,103.18
Receipts:		
Dues for 1926	$ 5.00	
Dues for 1925	2,545.00	
Dues for 1924	215.00	
Dues for 1923	105.00	
Dues for 1922	40.00	
Dues for 1921	10.00	
Dues for 1920	5.00	
Life Membership Dues	50.00	
Additional Life Membership Dues—Two Subscriptions at $ 50.00 each	100.00	
Donations	3,000.00	
Interest on Bank Balances	280.28	
Journal Subscriptions on Books Prior to February 15, 1922	48.88	
Advertising	572.33	
Sale of Back Numbers	224.90	
Sale of Colorimetry Reports	18.55	
Total Receipts		7,219.94
		$15,323.12
DISBURSEMENTS:		
Printing and Stationery	$ 162.77	
Postage and Incidentals	82.80	
Expenses of Annual Meeting	239.06	
Association of Scientific Apparatus Makers of United States Contract	4,800.00	
Total Disbursements		$ 5,284.63
CASH ON HAND, DECEMBER 31, 1925	10,038.49	
		$15,323.12

I certify that the above report is correct to the best of my knowledge and belief.

ADOLPH LOMB
Treasurer.

Adolph Lomb, Treasurer,
Optical Society of America,
Rochester, New York.

Rochester, New York.
January 29, 1926.

Dear Sir:

In accordance with your request, we have audited the books of the Cptical Society of America for the twelve months ended December 31, 1925.

We hereby certify that the above Statement of Cash Receipts and Disbursements is a true statement for the period from January 1, 1925 to December 31, 1925.

Respectfully submitted,
NARAMORE NILES, & CO.
Auditors.

The Principles of Physical Optics. By Ernst Mach, Late Professor of Inductive Science at the University of Vienna. Translated by John S. Anderson and A. F. A. Young. xi+324 pages, including 10 plates and 280 diagrams. E. P. Dutton and Company, New York. $6.00.

It is the author's purpose, as stated in the preface to present "the origin of the general concept of optics, and the historical threads of their development" and "to show, from a critical and psychological standpoint, how the ideas concerning the nature of light have been moulded at the hands of prominent individual workers, and what transformations these ideas have had to undergo on account of the revelation of new facts . . . ." The book is, then, essentially a history of Optics. But it is more than that, for the author has discussed, in places extensively, the development and interpretation of various experiments and theories, and has thus introduced a philosophical element of unique importance. A treatment of this kind should prove of great value to the student, and particularly to the teacher, who wishes a critical-historical survey of the subject, but who has neither time nor opportunity to carry out such studies. The numerous references to papers of great historical value will be appreciated by those who wish to refer to original sources. It must be pointed out, however, that the emphasis is on the optical theories, rather than on optical instruments, although brief treatments of the more important instruments are given.

An important feature of the book is the excellent collection of portraits of the earlier workers such as Kepler, Newton, Grimaldi, Young, Malus, etc. And in this connection it might be pointed out that brief biographical sketches of these men would have been appropriate.

The book deals entirely with classical optics. One does not even find "Quantum" or "Relativity" in the index. The latter *is* mentioned in the preface, but the author inquires "Will it prove to be more than a transitory inspiration in the history of this science."

It is predicted that the book will find a welcome place on the reference shelf of students of the subject.

F. K. RICHTMYER

Quantum Principles and Line Spectra.—By J. H. Van Vleck, Assistant Professor of Physics, University of Minnesota, 316 pp., $3.00. National Research Council, Washington, 1926.

This excellent bulletin will prove extremely useful to all who are interested in atomic physics. The title gives but little idea of the broad scope it covers. It contains a wealth of material which has not been treated in any textbook on the quantum theory, and references to the literature are very complete. While the results of fundamental experiments are quoted in profusion throughout the book, details of experimental procedure are subordinated; where theories are in conflict, and where data are inconclusive, the author frequently suggests experiments which would clear up the situation.

The various formulations of the quantum conditions, and the limits of their applicability are discussed in Chapters II–IV, while Chapter V gives illustrations of their applications. There are excellent sections on the principle of mechanical transformability. A more detailed treatment of the mathematical technique is reserved for a later chapter in which the fundamental theorems of Hamiltonian dynamics and perturbation methods of quantization are treated in a very readable fashion.

The chapter on the quantization of neutral helium is authoritative, and that on "the apparent non-conformity of quantum orbits to classical dynamics in atoms with more than one electron" is a valuable summary of a very controversial subject.

The author's treatment of the "correspondence principle" is refreshingly clear. He takes pains to point out that certain asymptotic connections between quantum theory and classical dynamics can be definitely proved, while other connections are only postulated. Thus, he distinguishes carefully the correspondence *theorem* for frequencies, and the correspondence *postulates* for intensities and polarization.

The sections on virtual oscillators and quantum theories of dispersion summarize a subject which is difficult to grasp because of its hypothetical and unsettled character. Many readers will not agree with the author's conclusion that "Kramer's dispersion formula, based on virtual oscillators, furnishes by far the most satisfactory quantum theory of dispersion." Without detracting from the service rendered by those who have made ingenious attempts to obtain a rational description of dispersion phenomena, the reviewer believes that a final solution cannot be achieved until we have a much more thorough knowledge of the dispersion curves of monatomic gases and vapors.

The whole book is surprisingly up-to-date. Even the theory of spinning electrons and matrix dynamics are touched upon. It is to be hoped that this report will run through many revised editions as quantum theory progresses, for it fills a real need.

Incidentally, many physicists would appreciate the opportunity of buying National Research Bulletins in a more durable binding.

ARTHUR EDWARD RUARK.

INSTRUMENT SECTION

THE EINTHOVEN STRING GALVANOMETER, A THEORETICAL AND EXPERIMENTAL STUDY

PART 2

By Horatio B. Williams

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PART 2[1,2]

SECTION 1. GENERAL PRINCIPLES OF CONSTRUCTION

Certain of the operating characteristics of the string galvanometer, such as short period, good damping, stability of zero under constant string tension, are common to all instruments of this type however crudely they may be made. Others, such as (a) stability of zero and of the plane of deflection under varying degrees of string tension, (b) equality of successive deflections for successive equal increments of current through the string, (c) capability of adjustment to very great sensitivity, (d) freedom from disturbance by extraneous vibration, (e) sharpness of optical definition, (f) amount of transmitted light available for photographic registration, etc., are distinctly dependent on design and construction. These matters concern the prospective user as well as the designer and maker, and an understanding of them is necessary if one is to make an intelligent choice from a number of available types of an instrument suited to fulfill a given requirement. Essential parts of all string galvanometers are the magnet, the string mounting, the microscopes and the string. Before the special problems

[1] The theory of the string galvanometer is treated in Part 1, this Journal, *9*, pp. 129–74, August, 1924. I am indebted to Professor H. W. March, of the Department of Mathematics, University of Wisconsin, for bringing to my attention inconsistencies in the statement of the solutions (10) and (16) of Part 1. The correct statement of these solutions is given in Appendix A, Part 2, where will also be found corrections of other errata in Part 1.

[2] When Part 1 was written the writer was unaware of a treatment of the theory of the Einthoven galvanometer published in English in 1914 by Dr. L. S. Ornstein. See Section VII, Part 2, for comment on this paper.

of string galvanometer design can be appreciated it is necessary to have some familiarity with these parts and their functions and we shall proceed to discuss their more general features.

A. THE MAGNET

1. *General Remarks.* The string galvanometer may be regarded as a special form of moving coil instrument and it was actually from this standpoint that it was developed (1). The strength of the magnetic fields is one of the factors which determines the sensitivity of a moving coil galvanometer, but it is well known that the sensitivity of the ordinary type of moving coil instrument cannot be increased indefinitely by augmenting the strength of the fixed field. It is a matter of great difficulty to make a coil without contamination of the wire or insulation or both with paramagnetic material and unless the distribution of this material is uniform and the field homogeneous, there will result an actual diminution of sensitivity when the field strength exceeds a rather moderate value. The moving system of the string galvanometer, consisting of a single tenuous conductor with no insulating covering, is less liable to accidental contamination. The strings are made of material which initially is not likely to contain significant traces of magnetic material. It is customary to employ field strengths of 20,000 gauss and more. The moving coil galvanometer can be made more sensitive by reducing the elastic restoring couple of its suspension, but a rather definite limit is set by the difficulty of maintaining a stable zero when the restoring force is made very small. It is usual to attain greater sensitivity by increasing the number of turns in the moving coil. In the string galvanometer this means of increasing sensitivity has been deliberately foregone and the coil replaced by a single stretched conductor in order to secure a very short natural period. The possibility of employing a field as strong as 20,000 gauss is of some assistance in regaining sensitivity, but the gain from this source corresponds to the use in a D'Arsonval type of a coil of ten turns in a field of 1000 gauss. That the string galvanometer can be at once sensitive and of short period is possible because the inertia of the moving part is very small and a small restoring force will suffice. Also *the zero position of a stretched string is very stable even when the tension is small.*

Although the field should be strong, it is usually not advisable to attempt to reach such field strengths as have been attained with large electromagnets in recent years. These magnets are designed to be excited with powerful currents and a slight rise in temperature is

usually immaterial. Sometimes they are wound with insulated copper tubing for continuous water-cooling and the amount of energy dissipated may be very large. In a measuring instrument precision must take precedence over sensitivity. The strings of a string galvanometer are often made of material whose coefficient of expansion differs greatly from that of the supporting framework and change in temperature of the instrument will affect its sensitivity. The field strengths which may be attained without great initial expense and the dissipation of large amounts of energy in the field coils are so large that they could hardly be doubled by the most elaborate and expensive construction. It is certainly not worth while to multiply the cost of construction and operation many fold for a twofold increase of sensitivity.

The desirability of concentrating the windings in the neighborhood of the airgap when intense fields are to be produced is now well known (2), (3). On the other hand if the poles are large the concentrated winding makes a coil whose average length per turn is great. The magnetizing effect is proportional to current and turns, the energy expenditure to resistance multiplied by current squared. Fields of very considerable intensity may be produced with comparatively small energy expenditure by distributed windings if the design of the iron frame is such that the permeability is high when the flux density is sufficient to maintain the desired flux in the gap. The only drawback is the somewhat extensive stray field which may affect other instruments.

2. *Lateral Uniformity of Field.* It is evident from the theory that the field must be uniform in the direction of the string's motion if the deflections are to be proportional to the strength of the current flowing in the string. As the string galvanometer is used almost entirely as a deflection instrument it is highly desirable that the law of deflection should be a linear one. The necessary uniformity can be secured by care in making the pole faces approximately parallel plane surfaces. The accuracy necessary to secure proportionality over more than a very limited range is really considerable. Failure to meet this requirement is one of the most common faults in commercial string galvanometers. Einthoven's original instrument, which we shall have occasion to mention frequently and shall hereafter refer to as the Leyden Model, gives deflections proportional to the strength of current for ten centimeters either side of zero with a magnification of 660 diameters and at a current sensitivity of about 10^{-7} ampere/cm. The replica of this instrument in the physiological laboratory of Columbia University

gives a similar result, but with a magnification of 900 diameters. Most commercial instruments have a much smaller range of linear proportionality.

3. *The Perforation for the Optical System.* This should be no larger than necessary, though its effect has been somewhat overestimated. As remarked by DuBois, (4), a considerable internal leakage occurs from the edges of the opening and the field in this central region may be of considerable intensity. The size of the perforation is detemined by the limitations of the lens maker's art. The lenses must be of high aperture, but it is not necessary that the pencil of light should be as large as that transmitted by a standard microscope objective of the same aperture. It would be possible theoretically to reduce the diameter of the lens system very materially, but this would entail expensive changes which the manufacturing optician could probably not afford to make considering the limited market for them. Up to the present time the makers of string galvanometers have contented themselves with standard microscope objectives mounted in the smallest metal case which would accommodate them. The effect of the perforation will be discussed in detail in Section IV of the present paper.

4. *The Length of the Airgap in the Direction of the Lines of Force.* In the Leyden Model this dimension is two millimeters and in most of the commercial models it has been the same or greater. The principal reluctance in the path of the flux is this airgap. It is much the largest factor in determining the necessary magnetomotive force and thus in great measure determines the size of the magnet. The weight of the Leyden Model is 100 kilos. For some purposes lighter instruments are desirable. One of the new designs described in Section 2 has an airgap of one half mm. There are, however, several disadvantages to be considered in connection with the reduction of the length of the airgap. First the difficulty of securing a linear deflection law is increased. A slight departure from accurate parallelism of the pole faces will have a more marked effect on the distribution of the field. This difficulty is not too great to be overcome by care and mechanical skill. There remains the danger that dust, particularly particles of iron may gain access to the gap and, becoming attached to the pole faces, touch the string and impede its movements or damage its coating. Experience has shown that with reasonable care an instrument with a half mm gap can be operated successfully, but the gap must be carefully inspected each time it has been opened to change strings, etc., to make sure that it is free from foreign matter.

5. *Necessity for Mechanical Rigidity.* There is a not inconsiderable tractive force between the polepieces[3] and this requires a construction of sufficient strength and rigidity to prevent deformation when the field is excited. It is usual to mount the microscopes and the parts that carry the string on the magnet. In connection with the mounting of the string it is necessary to take into account a molecular change, the magnetostrictive effect, which produces a small but significant deformation of the polepieces.[4]

6. *Shape of the Polepieces.* In order to concentrate the field in a narrow space the polepieces are made wedge-shaped. The angle included between the faces of the wedge was 90° in the Leyden Model and this angle has been adopted in many subsequent models. For the polepieces of large electromagnets in the form of truncated cones, Stefan, (5), showed theoretically that an included angle of about 110° is most favorable for producing a field of maximum intensity. DuBois, (4), has confirmed this result experimentally and shown that a difference of a few degrees is not critical. Salomonson (6) concludes that for a prismatic pole of a string galvanometer formed on the end of a circular cylinder 5 cm in diameter a top angle of 110° would be advantageous. DuBois (4) concluded that 90° would give the most intense field for a prismatic pole of infinite length. In the galvanometer later described as Type B this angle was made 120°. This was done as a matter of mechanical convenience and while it may have resulted in sacrificing some intensity, the practical result has been entirely satisfactory. DuBois has shown also that the greatest uniformity of field requires an angle smaller than that for maximum intensity. Uniformity is a prime requisite in the string galvanometer, but is required for only a small fraction of a millimeter in the middle region. Experiment shows that uniformity over the necessary extent can be attained even with the large angle provided the polefaces are made reasonably good approximations to plane parallel surfaces.

B. THE STRING MOUNTING

1. *General Remarks.* The sensitivity and period of the string galvanometer can be altered within wide limits by varying the tension on the string. In making this adjustment it is a great convenience if the string shadow remains in focus and retains the same zero throughout.

[3] The theoretical data for calculation of this tractive force are given by Maxwell, Treatise on Electricity and Magnetism, Edition of 1873, *2*, p. 255. A discussion of Maxwell's law is given by H. DuBois, The Magnetic Circuit, Translation by Dr. Atkinson, p. 167, 1896.

[4] See sub-section B, Article 2, of this Section.

It is possible to secure this condition, the possibility depending partly on the properties of the strings themselves and partly on certain adjustments. Assuming the string suitable,[5] the other necessary condition is that it shall be centered with considerable accuracy in the magnetic field. Though the field can be given uniformity in the direction of the string's motion for something less than a tenth of a millimeter either side of the middle of the poleface and midway between the poles, it will always be more intense close to the pole faces. All strings are either diamagnetic or slightly paramagnetic and by reason of slight contamination parts of a string may be paramagnetic and others diamagnetic. If the string is exactly centered its magnetic properties make little difference in its behavior, but if not it is likely that on deflection it will approach one or other of the pole faces with the result that its shadow will go out of focus. The same thing and probably a change of zero will result if the improperly centered string is slackened. These difficulties are readily avoided if means are provided for accurate centering. In the Leyden Model the clamps which receive the metal tips to which the strings are soldered are fastened to accurate and smoothly working cross-slides actuated by screws. There are two such slides, one at top and one at bottom and they move the one parallel and the other perpendicular to the polefaces. Devices of a less expensive nature have been tried with varying success. The results will never be altogether satisfactory unless the centering devices operate smoothly and precisely. Few instrument makers seem to have realized the need for precision in these devices and those who have never worked with a properly constructed string galvanometer in good adjustment consider the faults of the designer and builder to be inherent in the instrument. This is the more remarkable because the facts were set forth in detail by Einthoven (7, p. 314) in 1909.

2. *Tension Adjustment.* There must also be a fine adjustment for varying the tension on the string and this should operate smoothly and with a minimum of lost motion. When the tension on the string is small, a very small movement of the upper end will produce a relatively large change in tension (7, p. 308), so that an adjustment capable of close setting is desirable. An adjustment of the same general character as the wormgear fine focussing adjustments supplied on the best makes of microscopes is entirely satisfactory. A fine adjustment

[5] See sub-section D of this section, Article 4.

obtained from the Zeiss Works, of the type found on their large microscope stands, was used for the string tension adjustment on the Columbia instrument and has met every requirement.

3. *String Mountings.* There are two principal methods in use for mounting strings. The method employed in the Leyden Model in which the mountings are fastened permanently to the magnet and the strings inserted directly into the instrument is simple, substantial and effective, but requires of the operator considerable manipulative skill when strings require to be replaced. This type of mounting is entirely satisfactory for laboratory use where it will be handled by skilled personnel. In the other type the strings are mounted in a removable member carrying all or part of the centering and tension adjustments. This member often takes the form of a case in which the string is completely enclosed. The second plan presents itself in a considerable variety of designs. It offers obvious advantages when properly designed and executed, but on account of the difficulty of providing a number of precise adjustments in a compact removable member, it is particularly in instruments of this type that highly "simplified" adjustments are often found. It is essential that whatever plan is adopted shall secure practically dust-tight enclosure of the string when it is finally mounted and protect it against disturbance by air currents. Einthoven covered his entire instrument with a large box. This plan permits great simplicity in the construction of the instrument and has few disadvantages if, as is usual, the instrument is to be kept and used in one place. The same arrangement was adopted in the Columbia instrument with satisfactory results. If equally precise and robust adjustments can be provided in a construction which gives adequate protection to the string without requiring the covering of the entire instrument, it makes for somewhat greater facility in changing strings and permits freer radiation and convection of heat from the magnet coils. The instrument is also more easily moved about.

4. *Disturbance Due to Magneto-striction.* The simplest way to mount the parts that carry the string is to fasten them directly to the top and bottom of the polepieces. However, if the polepieces are massive, the magnetostrictive change of form may produce a slight tilting of the string mounting when the field is excited thus impressing upon the string a small angular component of tension. If the string is under considerable tension, the slight angular component thus introduced may produce no visible effect. If the tension is reduced to a value

comparable in magnitude with this component, the zero or focal plane or both will be altered on exciting the field.[6]

C. THE MICROSCOPES

An outstanding feature of the Einthoven galvanometer is that no dead load is carried by the moving part for the purpose of magnifying the motion. For this a microscope is employed. The optical system adopted for the original instrument is a Zeiss 4 mm apochromatic objective, of numerical aperture 0.95 as projection lens[7] in conjunction with a suitable projection ocular, such as Zeiss projection ocular No. 2 or No. 4. To illuminate the field a Zeiss 4 mm achromatic objective of N.A. 0.85 is used as condenser. For special purposes it is sometimes advantageous to use as condenser an apochromatic lens similar to the projection objective. These microscopes are mounted in borings in the polepieces and provided with focussing fine adjustments. In the Leyden instrument each microscope was provided with a double cross-slide for centering. If one is to realize the full advantage of an optical system of the quality above mentioned, adequate means for centering must be provided.

The design of the mounting of the microscopes is a matter requiring careful consideration. The usefulness of a sensitive measuring instru-

[6] The writer first observed this effect on a large instrument designed for the Rockefeller Institute for Medical Research. The effect was at first attributed to molar motion of the polepieces, although care had been taken in design and construction to insure against this. After it had been shown that the effect observed could not be so accounted for, its real nature was suspected and the suspicion confirmed by experiment. The question then arose why this effect had not been observed in the Leyden Model. In this model there is a large bronze plate through which the cores of the magnet pass and to which the polepieces are fastened. The support for the string mounting is fastened to the middle of this bronze plate where, presumably, the magnetostrictive change in the polepieces produces the least movement. In the Rockefeller instrument the effect was noticed then the tension on the string corresponded to a sensitivity of about 10^{-7} ampere/cm. It disappeared entirely when the tension was increased fourfold. In the Columbia copy of the Leyden instrument no effect of this kind was observed until the tension was reduced to an amount corresponding to a sensitivity of 10^{-8} ampere/cm. At this sensitivity the effect was well marked. The Rockefeller instrument was made to operate satisfactorily by bolting a bronze plate to the back of the polepiece with shelf-like projections above and below, but not touching them. The string housings were then mounted on these shelves. In subsequent constructions of the same type as the Rockefeller instrument the string housings were carried on a bronze fixture mounted to the polepieces with a Kelvin point-groove-plane arrangement which permits the greatest possible opportunity for change in the polepieces without strain of the string supports. This mounting will be illustrated in connection with the description of the instrument of which it forms a part. See Section 2, Article 2, of this paper.

[7] In correcting for spherical aberration the lens maker usually allows for the use of a coverglass. Lenses for use with a string galvanometer should be ordered adjusted for use without coverglass.

ment is greatly enhanced if it can be used in situations where there is much mechanical vibration without special precaution. A moment's reflection will show that the string itself will be but little influenced by mechanical vibration so long as it is under tension. Its inertia is so small that it will follow any movement of the frame of the instrument without appreciable lag. On the contrary the microscopes have a great deal of inertia and every joint in their mounting affords possibility for movement. Even solid metal is far from rigid. The microscope mountings should be designed for lightness with the maximum amount of rigidity and the minimum number of joints consistent with providing the necessary adjustments. Such joints as are unavoidable should be made with the utmost care. When the galvanometer is subjected to vibration there is a tendency for the microscopes to lag behind the movement of the frame and as the string does not lag, the relative movement is magnified to the same extent that the microscope magnifies the movement of the string in making measurements. If the galvanometer as a whole moved one tenth of a millimeter the movement would be just detectable on a photographic record. If, as is often the case, the magnification is 1000 diameters, a relative movement between string and microscope very much less than this is beyond the limit of tolerance.

When the string is mounted in a protecting case it is usually not practicable to use lenses of as high aperture as the 4 mm apochromatic on account of the very short working distance (about 0.2 mm) of this objective. Use of lenses of lower aperture imposes limitations as to magnification and possible speed of photographic registration which should receive careful consideration in the choice of a design. If extreme sensitivity or registration at high speed is required, the construction must be such as to permit the use of lenses of the highest available aperture. Achromatic lenses are satisfactory for subjective observation and for photographic registration at moderate speeds and magnifications. The Spencer Lens Company has recently made some special achromatic lenses for string galvanometer use with a view to good photographic definition rather than to excellence of the subjective image. These are less expensive than the apochromatics and give very good results. In some work optical filters may be used with advantage to improve definition.

D. STRINGS

1. *Quartz Strings, Chemically Silvered.* In some of his earlier experiments Einthoven tried strips cut from aluminum leaf (*1*, p. 1064). He

presently resorted to fibers of quartz glass (fused silica), coated with silver to make them conductors of electricity. The silver coatings were at first produced by one of the chemical reduction methods such as are used in silvering glass mirrors. These coatings are rough and tend to become more rough if the deposition is allowed to go on too long in one continuous reaction. The process may be controlled so as to diminish the roughness by silvering in two or more stages and by regulation of the concentration and temperature of the bath. The fibers may be polished by stroking them with a fine copper wire. Einthoven made excellent strings in this way and the process was described by Salomonson (8). The writer made many strings in this way and it is less difficult than might be imagined. A piece of copper wire about No. 40, B&S gauge, and 10 cm long is suitable. This should be fixed in a light handle such as a quill. One causes the wire to vibrate by slight motion of the wrist and under a shaded electric lamp brings the string stretched on a frame up to the vibrating wire in such a manner that the direction of its length is in the plane of vibration of the wire. The dark space under a table makes a good background against which to see the illuminated string. Strings of diameter as small as .0015 mm will withstand a surprising amount of lateral pressure, but care must be exercised at the beginning of the process that the wire does not catch on some of the larger roughnesses as the string is then easily broken. The polishing process is facilitated by securing a deposit of good quality, but it may take considerable labor to produce a really smooth and satisfactory string. Often the conductivity is lost after the first polishing and the silvering must be repeated. The coating should be examined from time to time with a lens to observe the progress of the work. Because of the probability that the silvering will have to be repeated it is most essential that the wire used for polishing be perfectly clean and never touched with the fingers.[8] Otherwise the string may be contaminated and new silver will not deposit firmly on a spot soiled with grease, etc. Fibers to be coated chemically should be silvered as soon as made and polished and re-silvered if necessary at once. If blank fibers are kept, even under cover, they quickly collect dirt and seem to undergo some surface change which makes it impossible to secure durable coatings. If fibers

[8] It is an excellent plan to brighten the wire by stroking it lengthwise with fine sandpaper and then electroplate it with silver in a bath of potassium-silver cyanide. After plating rinse the wire in distilled water and do not allow it to come in contact with anything except the string thereafter. It should be noted that in the silvering process the utmost cleanliness must be maintained and details followed with meticulous (almost ceremonial!) care.

are not polished the mass for a given conductivity will be greater than that of a smooth string. The shadow of a smooth string photographs more sharply than that of a rough one.[9] The conductivity of a chemically silvered string usually increases for a time due to condensation of the deposit and then diminishes. Failure to wash out all traces of chemicals hastens deterioration. The polished strings usually have a longer life, perhaps because loose metal is rubbed away and what remains is more adherent.

2. *Quartz Strings Coated by Cathodic Sputtering.* Much better strings can be made by cathode disintegration in vacuum (9), (10). Silver may be used in this process also and is the metal originally used by the writer. While it possesses certain theoretical advantages on account of its high conductivity and low specific gravity, the absolute amount of metal required is usually so small that gold may be substituted with advantage. When gold is used the excellence of the coatings is less likely to be impaired by slight accidental contamination of the vacuum with various vapors which are very prejudicial to silver. Strings made by this process last for years if properly handled and will even stand considerable abuse. One silver coated string was in use for five years in the Columbia laboratory. It was finally laid aside while still serviceable because a string of different properties was required.

3. *Strings Silvered by Evaporation of Metal at High Temperature.* The writer has also coated strings with silver by evaporation of the metal at high temperature (but below the melting point), in a very high vacuum. The process has the advantage that coatings of suitable resistance can be secured in from two to four *minutes* instead of the two or three hours required by the methods of cathode sputtering. However, the difficulty of producing and maintaining the necessary high vacuum sufficiently free from vapors which may react with the vaporized silver is very great. The difficulty is still greater if one employs an apparatus with ground joint which can be used repeatedly. Gold could not be used as it does not evaporate below its melting point. It seemed easier to develop a method for coating large numbers of fibers by cathode sputtering at one time and the method of rapid evaporation by heat was not further elaborated.

4. *Properties of Quartz Strings.* It may be of service to point out some properties of strings which are not generally known. In making the

[9] It is customary to use a cylindrical lens in front of the slit of the photographic recorder. If the string is rough, the reduced image of the shadow formed by the cylindrical lens will not have well defined edges.

blank fibers[10] it sometimes happens that a fiber will be observed to be distinctly curly. Such a fiber should always be rejected. It may be caught and stretched so that it looks well and it may be coated and used successfully provided it is always used under tension sufficient to keep it straight. If one tries to use it at such a tension that the tendency to curl can manifest itself, the zero and focal plane will be found to change with every alteration of tension. It may also go out of focus as it is deflected across the field. These are very annoying characteristics and limit the usefulness of the string seriously. Fibers may also be tapering, elliptical in section or ribbon-like. All such as well as curly fibers should be rejected. The value represented by a blank fiber is much less than that of the labor involved in coating, soldering and mounting in the galvanometer and incomparably less than that which the user will waste in trying to make measurements with it if he ever uses it slack.

Strings may acquire paramagnetic properties by contamination with iron dust. For this reason every care should be exercised to keep them protected from dust while in storage. After mounting in the galvanometer they should not be unnecessarily exposed. They seldom become magnetic while in the galvanometer, but the writer has seen this happen as a result of rubbing away a small rust spot on one of the polepieces while the string was exposed.

An otherwise good string may have its value much impaired in soldering it to the brass tips if through careless or unskillful manipulation it is given a torsion or is embedded in the solder in a direction which makes an angle with the direction the string will take when stretched. Einthoven pointed this out (*7*, p. 315), and indicated how the difficulty might be overcome by use of certain adjustments. The Leyden galvanometer and the one described in Section II, §2, as Type A are provided with all these adjustments. Type B, (Section II, §3) has one of these adjustments, but it is not altogether easy for the average user to make use of them. If care is used in making the strings it will seldom be necessary to attempt these unusual adjustments except in connection with very critical work at small tensions.

A good quartz string free from the above mentioned defects and centered properly in the field of a well made galvanometer can be slackened or tightened at will without showing change of zero or focal plane so long as it is under moderate tension. Even when the tension

[10] For the method of making blank fibers see Appendix G.

has been greatly reduced the change in zero and focal plane should be very slight and after re-focussing a very slack string it should be possible to deflect it entirely across the illuminated field without material change of focus and it should return to a practically constant zero, which, however, may not be quite the same as the zero under greater tension. Under any given tension the zero will remain constant within about one tenth mm, this figure referring to the shadow magnified 1000 diameters. To one accustomed to work with delicate torsional instruments this constancy of zero is one of the most striking features of the Einthoven galvanometer.

It may not be out of place to explain why so much stress has been laid on this matter of alteration of tension without change of focus and zero. The string galvanometer is used almost entirely as a deflection instrument and largely with apparatus for photographic registration. Its sensitivity can be altered and adjusted with great precision to any predetermined value in a small fraction of a minute provided the requirements above outlined are met. This is largely a matter of design and of care in providing good strings and as it will facilitate all work to be done with the instrument it is distinctly worth while. No one who has ever worked with a galvanometer possessing these good qualities will ever care to work with one that lacks them.

5. *Metal Strings.* Fine metallic wires made by the Wollaston process (11) may be used as strings. They have been largely employed in continental Europe. Those most generally used have been of platinum. No metallic wire yet made approaches the excellence of a good coated quartz string for most purposes. Before smooth strings coated by cathode sputtering were available some users of the string galvanometer preferred platinum strings because of their smoothness and the clear shadow they cast on the photographic surface when used with a recorder provided with a cylindrical lens. This distinction they no longer enjoy exclusively as the fibers coated in vacuum are quite as smooth as the best Wollaston wires. Platinum has a high specific gravity and a comparatively high specific resistivity. Its tensile strength is low. In order that the mass of a platinum wire shall be small enough to make it suitable for use as a galvanometer string it must be of very small diameter and it is then very frail. Platinum strings, at least those made from Wollaston wire commercially available, are always distinctly paramagnetic and as they are usually slightly curly and used at small tension they invariably in my experience change zero and focal plane on exciting the field in spite of the most careful centering. Notwith-

standing these drawbacks they have been much used and good work has been done with them.

Aluminum wires made by the Wollaston process were first used as galvanometer strings by the late Professor J. K. A. Wertheim-Salomonson of Amsterdam (12). Aluminum has a low specific gravity, is a good conductor of electricity and its magnetic susceptibility is about half that of platinum. It would be an ideal material for strings were it not for its low tensile strength and the fact tht it seems to have no particular elastic limit, but "flows" under tension. Strings of this metal will not last for any great length of time unless used at very moderate tensions. It is difficult to make them of very small diameter, but because of the low specific gravity of aluminum this would not be a serious objection and for some work might be an advantage. The really serious difficulty about their use is the yielding under tension. They may prove useful in some of the physical applications of the galvanometer where very short period is not necessary and low resistance is desired.

Gold wires are easily prepared by the Wollaston process. The susceptibility of gold (diamagnetic) is still smaller than that of aluminum, its specific resistivity is not great and its tensile strength is greater than that of aluminum, though less than that of platinum. It is less easily drawn into extremely fine wires than platinum and would be best used for work requiring low internal resistance and not very short period. Its specific gravity is high. Wires of copper and silver of such dimensions as can be drawn directly without resort to the Wollaston process may sometimes be serviceable as strings for special purposes such as use of the string galvanometer as a vibration galvanometer, tuned to the frequency of the impressed current. The alloy duralumin is light and has a tensile strength much greater than that of aluminum. I have never been able to get wires of this alloy of very small size. It is very stiff, but may be useful for special applications. Ductile tungsten is drawn commercially to a diameter of .01 mm. It has a high tensile strength, but is very stiff and has a high specific gravity. It was used as string material in the sound ranging galvanometers during the war. It curls badly and cannot be used except under tension sufficient to straighten out the kinks.

The metal glucinum, otherwise known as beryllium, is a ductile element of specific gravity 1.9. It is a good conductor of electricity. If metallurgists can produce this material in the form of fine wire it might be very useful as string material for some purposes, though I have no information as to its tensile strength or elasticity.

6. *Handling of Strings.* On account of their extreme tenuity quartz strings and fine metallic strings are not handled directly, but are stretched on small frames, usually of glass, on which they are coated (or corroded in the case of Wollaston wire).

From these frames they are transferred to brass tips by placing the tips in a suitable clamp which is held in a vise, while the string on its frame clamped in an adjustable stand is moved into position so that it comes into contact with the tips to which it is then soldered while still stretched on the frame. A string when stretched is easily visible to the naked eye when brightly illuminated. This is true even of strings so thin as not to be resolvable with a low aperture microscope. With practice in observing them in light of uniform intensity one can even make a rough estimate of the diameter. This depends on the intensity of the reflected light. If one end of a string becomes free while the other is held one gets occasional glimpses, but is not able to see the entire string at once. This is a principal reason for always manipulating them on frames. Wires as large as .01 mm may be handled like larger wires, but even these are more easily handled when stretched.

Strings are stored in holders provided with clamps in which the brass tips are fastened. From these they may be transferred to the galvanometer by means of a special clamp or manipulator which should have a simple tension adjustment. It is also a convenience if the ends of the manipulator are insulated from each other and provided with binding posts so that resistance measurements may be made while the string is in the device. In transferring a string the manipulator is clamped upon the brass tips before these are released from the storage clamps. It is then clamped in the galvanometer without releasing the manipulator clamps which are unscrewed after the tips are firmly fastened in the galvanometer.

7. *Repair of Strings.* Coated quartz strings if accidentally subjected to too great a tension may lose their conductivity. This is usually due to separation of the coating from the solder at the joint. By slackening the string in a manipulator and warming the solder gently with a small copper, it may be drawn further into the solder. In this work it is well to hold the manipulator in a vise and its binding posts may be kept connected with a bridge so that the conductivity may be tested at each step of the operation. If conductivity is not restored by the first attempt, slacken again and draw in the other end. This will usually be effective and the final resistance should not differ much from the value before the accident. These fibers are usually soldered with the alloy

known as Wood's metal which melts at 70°C. The copper used in soldering should be heated over a flame until the water of condensation evaporates again. It will then be hot enough to melt the alloy and if of small size it will cool rapidly which will facilitate its withdrawal, leaving the string held by the solidifying metal. The useful life of a string may often be greatly extended by repair in this manner.

SECTION 2. DESCRIPTION OF INSTRUMENTS

1. *The Leyden Model.* The Leyden Model is fully described in Einthoven's paper (7) on construction which may be consulted for additional details. The principal features are an electromagnet consisting of two iron cores 8 cm in diameter and 31 cm long, connected by a heavy yoke of rectangular section 29.5 cm long, 14.8 cm high and 6 cm thick. At the end opposite the yoke the cores pass through holes in a heavy bronze plate to which the polepieces are fastened. The polepieces, also of rectangular section, are 12.5 cm high, 6 cm wide and 8 cm long exclusive of the wedgeshaped ends. The top angle of the wedge is 90° and the polefaces are 2 mm wide. The airgap is 2 mm long measured in the direction of the lines of force. The polepieces are bolted to the cores as well as to the bronze plate and the wedgeshaped spaces between them are filled with snugly fitted brass wedges. The rear wedge is held in place by the bronze plate and that in front by two heavy bronze bars. The polepieces are thus firmly braced against mechanical stress produced by the magnetic field. The cores are wound with 1843 turns of copper wire 2.2 mm in diameter, the resistance of the coils in series being 2.6 ohms. Originally this instrument was provided with leveling screws, but these were abandoned as it was found that notwithstanding the fact they were well fitted the slight play magnified the effect of mechanical vibration ten fold. Other features of this instrument have been mentioned already. In point of convenience of operation, precision and sensitivity this original instrument left little to be desired. It has, however, one disadvantage: it is essentially difficult and expensive to construct.

2. *New Design, Type A.* In 1916 the writer described at a meeting of the American Physical Society in New York City, a new construction of the string galvanometer.[11] The accompanying figures, 1, 2 and 3, are

[11] The design of this instrument was undertaken at the request of the Rockefeller Institute for Medical Research when the coming of the war in 1914 prevented them from procuring the contruction in Europe of a copy of the Leyden Model. The re-designing was done to reduce the expense of construction. The form of the magnet was suggested by that of the older half-string magnet of DuBois (13).

from photographs of this instrument, which for brevity will hereafter be referred to as Type A. In the Leyden Model soft Swedish iron was

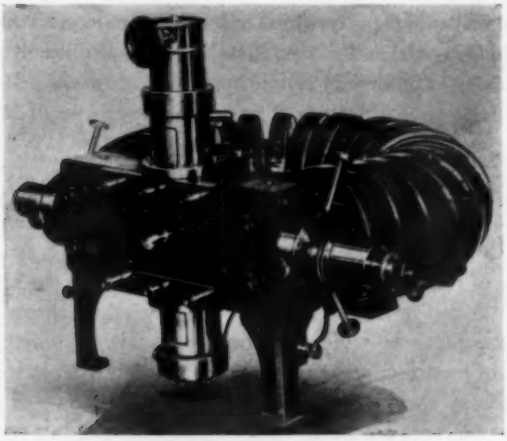

FIG. 1. *Type A. All parts in place.*

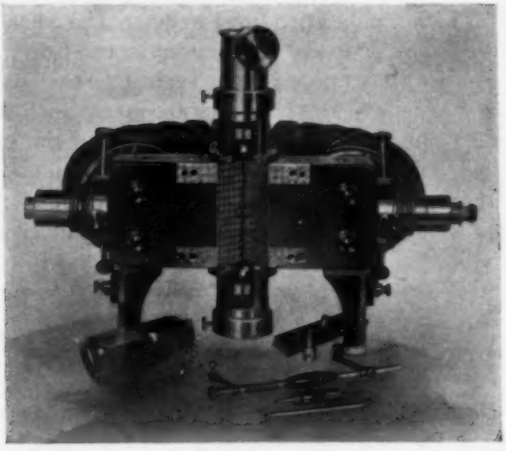

FIG. 2. *Type A. Front bracing bars and wedge removed. String housings open. Device used to insert strings is shown lying in front of the instrument.*

used for the entire frame of the magnet. In Type A the yoke is a casting of special low carbon steel of high permeability. In the first instruments of this type Swedish iron was used for the pole pieces

When Swedish iron of really high grade is available, it is a slight advantage to use it for pole pieces, but much Swedish iron contains seams of oxide introduced through folding in the forging of the blooms and these seams materially lower the permeability. It is also very tough and difficult to work so that its use adds to the expense of construction. Some instruments made with polepieces of this iron were no better than others made entirely of the soft cast steel. The diameter of the cross-section of the yoke is 8 cm, the same as that of the cores of the Leyden Model. The polepieces are of the same height and width as those of the Leyden Model, but longer to fit the milled-off ends of the

FIG. 3. *Type A. Rear view showing method of mounting string housings.*

ring-shaped yoke. The airgap in most of these instruments has been 2 mm, as in the Leyden Model, but a few have been made with 1.5 mm airgap. These give greater field strength for a given exciting current and offer but little more difficult in the matter of inserting strings, really presenting no difficulty at all to a skilled technician. The exciting coils are of No. 11 B&S gauge copper wire, of which there are 1920 turns. This wire is wound in a series of flat coils. The coils are wound on forms, then taped and connected in series after being strung on the yoke. The resistance of the coils in series is 2.5 ohms.

The strings are mounted in cylindrical housings which can be seen clearly in the figures. The clamps which hold the string tips are visible

in Fig. 2, which shows the instrument with the front wedge removed and the string housings opened. The string tips can also be seen, and lying on the table before the instrument is the manipulator used in removing and inserting strings. The clamps which hold the string tips are adjustable parallel to and perpendicular to the polefaces, by screws, two of which show in the figure. The upper housing is taller and contains in its upper part the adjustment for altering the tension of the string. This adjustment consists of a screw driven by a worm and gear and opposed by a spring. The airgap is closed in by brass wedges as in the Leyden Model. In Fig. 2 one of the wedges is lying on the table and in Fig. 1 it is shown in place. These wedges are scraped and ground to a close fit. Cylindrical gates which rotate in the housings complete the closure, the arrangement being evident in the figures. Besides the opening in the front of the housings there is one at the side also closed by the rotation of the gate. This admits the screwdriver when a new string is to be clamped in place and also permits the operator to view the string laterally so as to judge of the centering in the antero-posterior direction. In observing the lower end a small mirror is used for the lateral view. When the wedges are in place and the gates closed the string is well protected against dust and air currents, so that it is not necessary to cover the entire instrument.

The polepieces are braced in rear by a small bronze plate which fits between the butt ends of the iron ring and is screwed to the polepieces. In front are two bronze bars which further brace the instrument and hold the front wedge in place. Both string housings are mounted on a massive bronze carrier, the back of which can be seen in the rear view, Fig. 3. This carrier is held against the rear bronze place by heavy bronze spiral springs and is located by adjustable screws whose tips rest in a point, groove and plane respectively, the well known arrangement of Lord Kelvin. The upper and lower plates of this carrier do not come in contact with the top and bottom of the polepieces, but have a slight clearance. Felt washers let into these plates make a dust-proof joint and prevent disturbance by air currents.[12]

The microscope tubes slide in sleeves bored and lapped individually to as close a fit as will permit the necessary freedom of movement. The sleeves have a shoulder which fits in a centering box provided with three adjusting screws. The fit of the shoulder in this box is made so close that the friction will hold it in position even if all the adjusting screws are loosened. There are no gibs or springs and when one screw is

[12] In connection with this carrier see footnote 6, Section 1-B, Article 4.

loosened another must be tightened before movement will take place. A centering box so made is incomparably more rigid than the usual constructions and adjustments can be made with it quite as easily. The adjustment once made is far less liable to accidental change than is the case where easy fits and the use of springs and gibs is adopted. No coarse adjustment for focussing is needed. For fine adjustment a cap fits over the microscope and against a shoulder on the tube, a ball bearing thrust collar being interposed. The tube is pushed out by a coiled spring made large enough to slip freely over the tube. The cap screws to a threaded collar on the sleeve. The screw threads of this adjustment are carefully made on a thread-milling machine and in spite of the large diameter they function smoothly. Adjustable screw stops are provided so that the microscopes can never be screwed in far enough to touch the string. They require to be adjustable as the lens mountings vary somewhat in length. When once adjusted no further change is needed unless the lenses are changed. As the mountings of objectives vary in length, care should be taken to see whether the stops require readjustment if lenses are changed.

In the Leyden Model there was an opening from the airgap along the microscope tubes which required to be packed lightly with soft paper to prevent disturbance of the string by air currents. This opening is effectually closed in Type A by the manner of mounting the microscopes.

The entire instrument stands on three legs without leveling screws. The instrument has been used to record the human electrocardiogram in a factory building where a large number of machine tools were in operation, without any disturbance from mechanical vibration appearing in the photographic records. The instrument stood on an ordinarily substantial wooden table, but no other precaution against disturbance was taken except the constructional features of the instrument itself already mentioned. Besides the rigidity of the microscopes the power of the optical system and the strength of the magnetic field contribute to the result as the string is tighter for a given sensitivity than it would be in a less powerful instrument. Special emphasis should be laid on this comment as the shortcomings of inferior designs have led to a widespread opinion that the string galvanometer is especially susceptible to disturbance by vibration and requires elaborate precautions to protect it. The reverse is true of a good instrument and *there is probably no instrument comparable in current sensitivity with the string galvanometer which is so little affected by vibration.* Type A was planned

for laboratory use, but on account of the difficulty of obtaining satisfactory instruments for clinical use during and immediately following the war years a number of these large galvanometers were supplied to hospitals and clinics. In the spring of 1919 the maker[13] suggested that the design of a new model be undertaken with a view to providing an instrument suited particularly to the needs of physicians and capable of being manufactured at a lower cost than Type A.

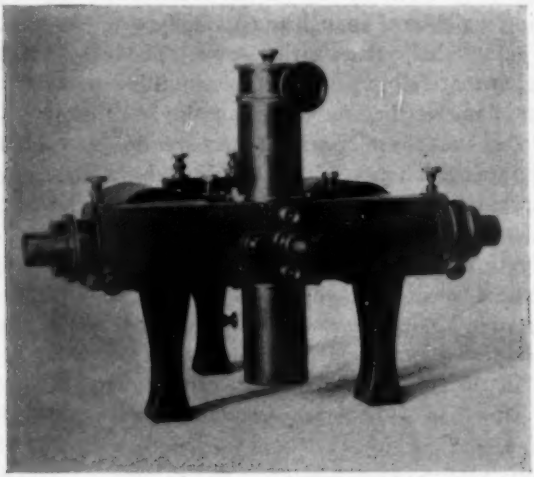

FIG. 4. *Type B. All parts in place.*

3. *New Design, Type B.* The problem of designing this new instrument had not been long under consideration when it was realized that it would be possible to make it not only a smaller and more convenient instrument for clinical use, but also a precise galvanometer of comparatively small size which would be capable of doing most of the work for which the large galvanometer is suited and even better adapted for some applications. Figs. 4 and 5 show this instrument, the design of which was completed in the spring of 1920. The result desired in Type B was an instrument of smaller weight and less cost, but of quality equal

[13] These instruments were made by Mr. Charles F. Hindle. I take this opportunity to acknowledge my indebtedness to his mechanical skill and to express my appreciation of his conscientious efforts to meet every requirement. Though the writer is responsible for the essential features of the design of these instruments, even to many of the minor details, much was left to the discretion of Mr. Hindle and he has initiated various improvements. The manufacture is now carried on by the Cambridge Instrument Co., Inc.

to that of Type A. It was desired to retain the same optical system and at the same time make such arrangements as to minimize the special skill required of the operator.

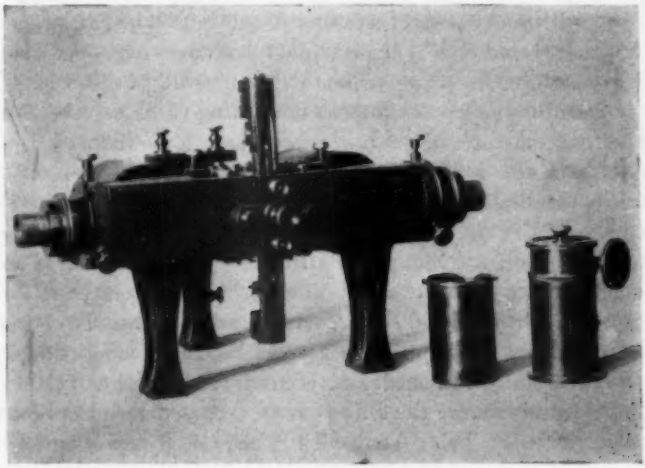

FIG. 5. *Type B. Upper and lower string caps removed, string carrier in place.*

The size of the magnet is determined by the dimensions of the airgap and the field strength desired. The height of the airgap depends on the string length. If strings were perfectly flexible and if it were not necessary to perforate the magnet for the optical system, the obvious

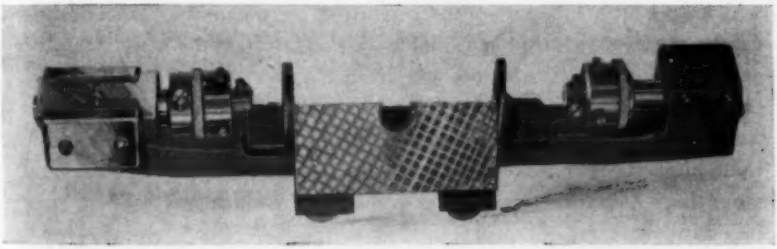

FIG. 6. *String carrier for Type B.*

way to make a small galvanometer would be to reduce the length of the string indefinitely. Previous experience with a galvanometer hastily constructed in the writer's laboratory some eight years before to meet

a special requirement had provided the information that a pole length of six cm was not too short for good results and this length was chosen for Type B. Another factor which affects the size of the magnet is the leakage flux. With high flux density in the airgap this is certain to be large and with a distributed winding it will be somewhat larger than with a concentrated one. The principal reluctance to be overcome is in the airgap and the shorter we make this the less will be the coefficient of leakage and the smaller the number of exciting turns may be. With a short gap the diameter of the iron cores may also be reduced.[14]

Apart from the reduction in height of the polepieces, the chief feature in the design of Type B which makes possible reduction of size and weight is the length of the airgap which has been made one half mm. The construction of the string mounting is shown in Fig. 6. It is made in a single removable unit containing the centering devices and also the spring and slide of the tension adjustment. To center the strings this mounting is put into a special fixture made of hardened cast steel, carefully seasoned and ground to dimensions with great precision. In this fixture the string can be centered with the necessary accuracy, the work being controlled with a low power microscope provided with an ocular micrometer. Guides are provided on the mounting which fit corresponding parts on the magnet so that the centered string can be inserted without danger of coming into contact with the polepieces or the lenses. The microscopes are centered on the string and with each other. By the use of suitable jigs, fixtures and gauges in manufacture it is possible to make the magnets so nearly alike that when the microscopes have once been centered on any string which has been located in the mounting by means of the centering fixture, any other string centered in this fixture may be inserted in the galvanometer and its shadow will at once appear at or near the center of the illuminated field. This is an advantage to the user who does not wish to incur the risk and trouble of handling strings and centering them himself. It is also convenient if one wishes to keep at hand several mountings containing strings of different resistance, etc. These may

[14] It was part of the original plan of this paper to give in some detail the calculations on which the design of this magnet was based. The paper has grown to such size that it seems best not to expand it unnecessarily and it appears probable that anyone who would be likely to make use of magnetic data in design would already be fairly familiar with the subject. An excellent practical treatment is to be found in "The Magnetic Circuit," by V. Karapetoff, McGraw-Hill Book Co., New York, 1911. DuBois' "Magnetische Kreise" will provide additional data and bibliography.

be interchanged in a fraction of a minute. When not in the galvanometer the mounting and string can be kept in a dustproof metal box in which it may also be shipped conveniently. The construction is such that the skilled technician may insert and center his own strings while those who prefer may have this service performed for them by the maker. When the mounting is in place the lower end is covered by a cap provided with a bayonet joint. The upper end is covered with a housing which carries the wormgear and screw of the tension adjustment. The microscopes are similar in all respects to those used in Type A, except that it has been possible to reduce the tube length to 160 mm. In Type A the tube length is 250 mm so that objectives corrected for use in the one type should not be used in the other.

The magnet frame consists of two similar castings of high permeability steel, cores and polepieces being cast in one piece. There is no separate yoke, but the cores curve around behind and terminate in flanges which bolt together in the midline opposite the airgap. There is thus but one joint in the iron. The magnet coils are disposed similarly to those in the Leyden Model, but the legs are separated further. There are 1000 turns in the exciting coils equally divided between the two legs and their resistance in series is 1.5 ohms.

4. *Other Models.* It would require much space and some information not at the writer's command to describe all of the various constructions which have been made either privately or commercially. We shall mention a few which are important either from the large number in use or because of their interest in connection with the development of this instrument. Dr. Einthoven has developed several types since the construction of his original model[15] and these he will probably describe soon. One very interesting model has been described recently by his son, W. F. Einthoven (14). In this instrument the strings are extremely short and are mounted in the space between the lenses so that the field available for operation is merely the leakage into the region of the perforation. It is, nevertheless, sufficient.

The late Dr. J. K. A. Wertheim-Salomonson designed and built privately several string galvanometers. In these instruments the windings were concentrated and the iron so arranged as to give two parallel magnetic circuits. The string is mounted on a removable member. Another model with concentrated windings was designed by Dr. George Fahr for Professor L. Asher, of Berne. An instrument based on this

[15] Oral communication.

design, but considerably modified, is built commercially by Stoppani of Berne. Boulitte, of Paris, builds a simple and compact string galvanometer designed by Mr. Lucien Bull, of the Institut Marey. This instrument is developed from the small instrument designed by Mr. Bull during the War as a part of the Bull-Tucker sound-ranging equipment. I have not seen this instrument and know it only from the catalog of the maker. It seems to lack certain adjustments which I regard as essential in an instrument adapted to general service, for example I can see no provision for the accurate centering of the strings. Presumably it has been found serviceable for the work it is designed to do, clinical electrocardiography.

The Cambridge Instrument Company, Ltd., of England, has made a large number of string galvanometers after a design by the late Mr. W. Duddell. In this instrument the string is enclosed in a thin brass box provided with thin mica windows at the center. The box hangs on a point-groove-plane support which permits it to be adjusted in the airgap and there is a screw adjustment in a direction at right angles to the direction of the lines of force. The enclosure of the string offers obvious advantages, but as the lenses cannot be brought close to the string, their possible aperture is limited. The usual equipment is a 16 mm apochromatic objective of N.A. 0.30 as projection lens. It is possible to use an 8 mm objective of N.A. 0.65 by bringing the string close to one side of the box, but this can be done only if the string is to be used under tension sufficient to keep it taut. The advantage apart from better optical definition is that at the higher magnification thus made possible the range of proportionality of the instrument is increased. This instrument is also supplied with a box, or "vibrator case," containing two strings which may be projected simultaneously on the same photographic registering device. This is useful for recording two related trains of events, such as current and voltage. In the original design there was a single large exciting coil placed on the yoke opposite the airgap. This was not an ideal design from the standpoint of energy consumption and avoidance of leakage and the instrument is now made with the winding distributed in two coils on the legs of the magnet. This instrument is satisfactory for electrocardiography, for which it has been largely used, and for other applications where the string is under tension. It can be used with the string slack, but is less convenient in this condition than an instrument provided with screw adjustments for centering the string, as the centering by swinging the box is not so easily accomplished. On account of the method used for adjustment of the string in the box there is apt to be lateral and torsional

stress at the ends of the string which causes more change of zero and focal plane than occurs with the Leyden type or instruments based on it. The change of zero is readily compensated by the lateral adjustment screw and if the 16 mm lens is used there is sufficient working distance to permit of keeping the string in focus. The microscope mounting is a geometrical clamp which makes this instrument also very insensitive to vibration.

Edelmann of Munich was probably the first instrument maker to build string galvanometers commercially. His first three instruments were based on the Leyden Model and were very similar to it. He then designed an instrument with concentrated windings whose form was evidently suggested by the large Weiss electromagnet. A considerable number of these instruments came into use between 1907 and 1914. The string is mounted on a separate carrier with centering screws and a pinion and gear tension adjustment. The optical system was that adopted by Einthoven. The instrument is nicely made, but certain features of the design are not too happily chosen. Briefly the principal criticism pertains to lack of rigidity in the magnet frame and in the microscope mountings. The string carrier is also lacking in solidity. The magnet frame deforms on exciting the field, so as to change the alignment of the microscopes and destroy the parallelism of the pole-faces. This difficulty can be overcome by bolting across the top of the frame a rectangle of cast iron with one side removable to permit insertion and removal of the string carrier. The instrument was evidently designed with a view to exclusion of dust and air currents, but the means adopted are insufficient and the instrument should be covered with a box. It is decidedly sensitive to vibration on account of the manner of mounting of the microscopes. Removal of the leveling screws is a help and if necessary the entire instrument may be mounted on a Julius suspension. On account of its excellent optical system it is worth while to make these alterations. The writer has used an Edelmann galvanometer so altered with good results. Edelmann also constructed a large model with permanent magnets and several smaller models, some with permanent magnets and some with electromagnets. The latter heated badly and would not maintain constant sensitivity long enough to be used for serious measurements (15). The permanent magnet small type was lacking in sensitivity and had no considerable range of proportionality. The adjustments on these small instruments were extremely crude. There was also an Edelmann instrument with two strings in one magnetic field which I have not seen.

Another galvanometer designed for clinical use is by Huth of Berlin. This has a ring-shaped frame with polepieces projecting inward so as to give two parallel magnetic circuits. The windings are concentrated over the poles. The first instruments must have heated from the energy dissipated in the magnet coils as later instruments were equipped with metallic fin radiators embedded in the windings and projecting out for a considerable distance. The string mounting is detachable and provided with a cover which is automatically removed as the mounting is screwed into place. There is thus combined the advantage of the protecting box with the advantage of using the naked string so that lenses of high aperture can be employed. I do not know what lenses were provided. The instrument I saw did not give good optical results, perhaps because the lenses could not be centered. I believe this was supposed to be done once for all by the maker.

During the war an instrument designed by Mr. Lucien Bull was much used for sound ranging. Six strings were mounted in one magnetic field. These instruments were not used as current measurers, but as indicators of the time of beginning of a current. It was unnecessary to provide elaborate adjustments either for centering or for regulation of tension. The strings were mounted on a frame provided with insulating bridges and spacing pins and were fastened at one end to fixed pins and at the other end to pins which could be rotated like the keys of a banjo. The tension could thus be regulated to a suitable value and as the variations due to temperature change made no material difference the fact that the adjustment was not particularly convenient was not objectionable. This string mounting from its peculiar construction was aptly known in the Service as a "harp." These instruments were extensively and effectively used in the English, French and United States armies. The sensitivity did not require to be great and low power optical systems were used, the projection lens being a photographic objective giving a magnification of about thirty diameters. The six strings spaced about a half mm apart made a system whose magnified shadows were a considerable distance apart. Each shadow was intercepted by a small prism which picked up a small part of the adjacent pencil of light. By suitable adjustment of the six prisms, the six small pencils of light could be brought side by side on a strip of photographic paper some three cm wide and so aligned that simultaneous movements of the wires would fall on the same ordinate of the record. Such an arrangement has obvious physical and engineering applications.

Sanborn, instrument maker of Boston, has recently placed on the market a string galvanometer of small size as part of an equipment for clinical electrocardiography. The writer has seen one of these instruments, but has no data as to its constants or performance.

SECTION 3. DETERMINATION OF THE PHYSICAL CONSTANTS

1. *The Strength of the Magnetic Field.* The strength of the magnetic field may be determined in several ways. Einthoven (7) has employed a balance method as has Wertheim-Salomonson (6), who used also the method involving change in resistance of bismuth wire. The use of an exploring coil and ballistic galvanometer is an obvious method and may be applied in several ways. A variation of this last method would be the employment of a direct reading fluxmeter, but on account of the smallness of the airgap a coil of many turns cannot be employed and the ballistic galvanometer will give better results.

The curve of magnetization may be obtained from a number of observations by any of these methods at different strengths of exciting current. It may also be obtained as was done by Einthoven (7) by noting the increase in deflection of a quartz fiber kept at constant tension and carrying a constant direct current while the exciting current on the field coils is gradually increased from zero to the maximum value.

Either the maximum or the average field strength can be obtained by any of the above methods. The quantity H, which enters the equations of Part 1, is not the maximum but the *effective* field strength. By this we mean the strength of that uniform field in which the deflection of the middle point of the string produced by steady direct current would be the same as in the actual field. Einthoven has obtained a value of the effective field by measurement of the electromagnetic damping of a quartz string.

Referring to Eq. (6), Part 1, it will be seen that all the quantities are easily measurable except H and m. If m can be determined independently of H, the latter becomes known. It is not possible to determine with any great approach to accuracy the value of m for a coated quartz string from measurements of the diameter and resistance. For the purpose of determination of field strength it is not necessary that the inertia of the string shall be small as all the measurements except that of natural period are made in the state of equilibrium and large inertia in no way hinders determination of the period. If we substitute for the light string a metallic wire of suitable diameter, we may deter-

mine its linear density in advance by weighing a measured length, say a meter. We should, however, bear in mind that Eq. (6) was developed on the assumption that the string is, or is similar to, a perfectly flexible conductor. The quartz string approximates to this assumption so long as the tension is large in comparison with the force required to overcome rigidity. The metallic wire should be put under correspondingly great tension. In any event the rigidity will tend to make the measured value of the effective field smaller than the actual value. To carry out the measurement the wire is deflected by steady current whose strength is measured with a suitable instrument. The deflection of the shadow is measured and the circuit opened. The wire returns to the position of equilibrium after a long series of oscillations and while these are taking place a record of the motion is made on a photographic plate or film moving at the rate of at least a meter a second. Simultaneously there is registered on the plate a record of the time in thousandths of a second. If the tension is suitable, the period can be determined with sufficient accuracy without correction for damping, which, however, is easily made if desired. An independent determination of the magnification is necessary since the quantity U in the equation is the actual unmagnified motion of the string. Also if the deflecting current is measured in amperes a factor of 10^{-1} must be introduced, since I of the equation is in cgs units.

The possible errors of the several measurements give for the possible error of the calculated value of H a magnitude of one per cent. Considering the use to be made of the result an attempt to reduce this error would probably not be worth the labor involved. This method was devised[16] for an investigation of the Columbia University copy of the Leyden Model and was subsequently used for studying a number of the Type A instruments. The magnetization curves of these instruments were obtained by the method of observing deflections of a quartz string carrying constant current while the field current is varied. For one of these observations the field current is made the same as that used when the effective field is measured by the heavy wire method and thus the values of all points on the curve are determined. The best wire available at the time was some copper wire .048 mm in diameter. The fine tungsten wire now available would perhaps have been better. When this method of measuring the effective field was first applied it was the writer's opinion that there is actually a significant, if not a

[16] This method was described at the February meeting of the American Physical Society at New York City in 1916.

large, difference between the maximum and effective values. This question is perhaps of no great moment in connection with any particular instrument, but is of considerable consequence in the matter of design of new types and will be taken up again in Section IV. The amount of space devoted to description of the heavy wire method of measuring effective field strength is not justified by its importance except for the circumstance that measurements made in this manner have been used as an experimental control to test the validity of theoretical deductions.

2. *The Linear Density of the String.* The linear density, m, may be determined as soon as the field strength, H, and the undamped natural period, τ_0, corresponding to a particular value of the sensitivity, are known. If the motion took place in a vacuum no further comment would be needed. Actually the matter is not simple. In 1828 the astronomer Bessel, in connection with a report of a new determination of the length of the seconds' pendulum, pointed out the necessity of correcting for the inertia of air moved as well as for buoyancy. His experimental results led Stokes (16), (17) to an extended theoretical investigation which included consideration of the motion of a cylinder in a viscous fluid. Stokes arrives at the conclusion that the inertia and also the coefficient of mechanical resistance, k, will be enormously increased by the moved air when the radius of the moving cylinder becomes small. The theory predicted a very large effect for cylinders of small radius, but Stokes was inclined to doubt the accuracy of the predictions when the radius becomes very small. He thought the damping would be even greater than the theory would predict (17), p. 67. It appears from Stokes' work that the additional inertia will be greater the slower the motion.

In working out a method for correction of curves when the galvanometer is used overdamped, Einthoven (18) obtained results which led him to conclude that when the string moves slowly it behaves as if its mass were increased. It appeared to have different values of m during various stages of a single deflection. He also found this effect greater for a string of smaller diameter. The effects he observed were large, amounting in some instances to a six-fold increase of m. At the time of publication of these results (1906), Einthoven was probably not familiar with Stokes' work, as he does not mention it and speaks of the result as quite unforeseen. The interpretation he placed on certain of his observations led him to regard k, the coefficient of mechanical resistance, as constant over a wide range of tensions, and

in making the measurements from which the change of mass was inferred he determined k for a single point of the curve where it could be measured with greatest precision and assumed that this value held for all other points. Now Stokes' theory, as already mentioned, points to an increase of k and m *both* as the motion becomes slower. One striking experiment described by Einthoven is the following:[17] Two curves were registered showing the response of the string to sudden application of steady current. In the second record the tension on the string and the speed of the photographic plate were both increased ten fold. The current through the string was similarly increased so that the deflection in both records was the same. The shapes of the curves were almost identical. But this experiment really proves that the *damping*, $k/2m$, is constant, and in connection with Einthoven's other observations affords experimental grounds for belief that Stokes' theory is at least qualitatively applicable to cylinders of very small radius. Both these curves were registered at sensitivities greater than that at which variation of m was observed, so that *a fortiori* there must have been alteration of m between these two records. That being admitted, we must conclude that k also changed and the remarkable deduction to be made from these observations is that *the ratio between k and m remains constant.* Stokes' theory points to the conclusion that for a certain range of values of the radius of the cylinder such a compensatory variation of these two quantities might be expected. Stokes' paper is long and the mathematical analysis is difficult. It would require a very extensive digression to consider it in sufficient detail to permit introduction of numerical computations based on it for comparison with the observations on the string galvanometer. The writer hopes to have an opportunity to accumulate experimental data shortly for the express purpose of comparison with Stokes' theory and the results will be communicated later. As experiments of the writer, discussed in Appendix C, have shown constancy of the ratio k/m over a range of tensions between that corresponding to critical damping and a tension somewhat below the maximum safe limit, it may be concluded that the constancy of this ratio holds over the entire range of sensitivity to which the differential equations of motion apply and probably beyond that range since in Einthoven's experiment the lesser tension corresponded to a sensitivity of 10^{-9} ampere/cm.

The true value of m can be obtained from the data of a steady current deflection at a tension for which τ_0 has been determined. Rearranging

[17] (1), p. 1068 and Figs. 1 and 3, Plate III, and (18), pp. 503–04.

Eq. (6), Part 1, $m = HI\tau_0^2/32U$. It will be shown in the next § that the value of τ_0 can be uniquely determined. Further discussion will be found in Appendix F.

3. *The Free, Undamped Period.* The most direct method of determining this constant is to subject the string to tension sufficient to make its motion distinctly periodic and from a simultaneous photographic record of the motion and of a series of uniform time intervals, determine the damped period and the decrement. From these data the undamped period, τ_0, can readily be obtained.[18] If the string is short or sufficiently massive there is no difficulty in making the determination in this manner. Strings of small mass and of the length usually employed in the larger instruments, say 13.5 cm long and .001 mm in diameter, can hardly be made to oscillate in air enough to permit a determination of any considerable precision without subjecting them to a tension likely to be injurious. In Part 1, pp. 147-48, a method is given for determination of τ_0 when the string is critically damped. Another method applicable to the critically damped string is to impress sine wave alternating current, varying the frequency while the amplitude is kept constant by use of an auxiliary measuring device. The natural period agrees with that frequency of impressed current which gives a deflection half that produced by the steady current of corresponding value. For proof see the curve, p. 143, Part 1. If the source of A.C. contains harmonics, the determination is less sharp. A still rougher approximation may be obtained with great ease from any record of response to sudden application of steady current if damping is critical. If t in Eq. (15a), p. 147, Part 1, is put equal to τ_0, the numerical solution shows that the deflection will be complete to within approximately two per cent of the final value in a time after application of the emf equal to τ_0. If then we measure on a record the elapsed time from the beginning of the motion until it is 98 per cent completed, this will give a rough approximation to the natural period.

In Appendix B it is shown that $\tau_0 = \tau_k/\sqrt{1+\lambda'^2/\pi^2}$ where $\lambda' = k\tau_k/4m$. Substituting this value of λ'^2, we have

$$\tau_0 = \frac{\tau_k}{\sqrt{1+\dfrac{k^2\tau_k^2}{16m^2\pi^2}}}$$

from which it is seen that the value of τ_0 is unique, since it depends on the measured value of τ_k and the ratio k/m which has been shown to

[18] See Appendix B.

be constant. τ_0 is thus a true constant. This justifies the statement at the end of the preceding article that the value of m determined from a steady current deflection is unique and is the true value independent of the effect of the air in which the motion occurs.

4. *The Damping Coefficient, k.* This is the coefficient of mechanical resistance in dynes per cm/sec which acts on unit length of the string tending to annul the motion. When the string is a light coated quartz fiber, the air damping is usually so great in comparison with electromagnetic damping that no very great error will result from treating the problem as if the total damping followed the same law as the air damping. This was assumed for simplicity in Part 1, but the theory there developed cannot be applied without restriction to the motion of a string which is a good conductor if the resistance of the external circuit is small. See Section VII.

As mentioned in this Section, §2, k is not really a constant, though in a restricted sense it may be so regarded. It may be determined from the ratios of the successive amplitudes of a train of damped oscillations, the equation to determine k for the oscillatory condition being, $k=4m\lambda'/\tau_k$, for the development of which see Appendix B. If the string cannot be made to oscillate, determine τ_0 from a critically damped deflection, when k is given by $k=4m\pi/\tau_0$. It will be observed that m enters both equations. The real significance of k is in the damping $k/2m$ and experiment has shown this to be constant. If we introduce in either of these equations the true value of m we get a value of k corresponding to a state of motion such that not enough air is dragged with the string to affect the value of m sensibly. On account of the peculiar compensatory relation between k and m, we may use the true values in computation and predict the behavior of the galvanometer correctly at any tension because the changes are always of such a nature as to preserve the constancy of the ratio. The dimensions of k are $[MT^{-1}]$. The damping is of dimensions $[T^{-1}]$, that is, the damping is a time. It is the reciprocal of the time in which oscillations die away to $1/e$ of their initial value, or in which a critically damped deflection dies away to $2/e$ of its initial value.

SECTION 4. EFFECT OF PERFORATION OF POLEPIECES ON THE SENSITIVITY OF THE STRING GALVANOMETER

1. *Theoretical Considerations.* It has been a generally accepted view that weakening the field at the middle region must result in a significant diminution of the sensitivity of the galvanometer. This view seems as a result of our common notions to meet with instinctive approval. The

writer never questioned it until in looking over some of the work in Part 1 several months after its publication, attention was arrested by Eq. (23), p. 156. If U is the steady current deflection produced by the current, I, in a string immersed in a perfectly uniform field, then if everything remains the same except that the string is clamped fast at the points $5l/12$ and $7l/12$, this equation states that the new deflection due to the current I, is $U_2 = U/36$. The deduction of Eq. (23) in Part 1 leaves something to be desired in the way of logical rigor, but it can easily be improved. From Eq. (6) we readily obtain $U = HI\tau_0^2/32m$. The only factor in this expression which will be altered by change in the length of the string is τ_0. The period of a vibrating stretched string is $\tau_0 = 2l\sqrt{m/T}$, from which it is evident that τ_0 varies directly as l. If the length be shortened to one sixth, the natural period becomes $\tau_0/6$ and the square of the natural period becomes $\tau_0^2/36$, whence it follows directly that $U_2 = U/36$. Now if the field in the middle sixth be conceived to vanish entirely, the whole string being now free to move, it will be acted on by electromagnetic forces at all points except those lying between $5l/12$ and $7l/12$. The tension is uniform throughout and is taken as being so great that the increase of tension with deflection is negligible. The conditions at the points $5l/12$ and $7l/12$ are then entirely unaffected by the absence of electromagnetic forces over the middle sixth and in consequence the region from zero to $5l/12$ and that from $7l/12$ to l will be deflected to exactly the same places that they would be if the field were uniform throughout. The middle sixth then contributes only 1/36 of the entire deflection of the middle point and if the field vanished altogether in the middle sixth the deflection of the middle point would still be $35/36U$, or $.972U$. In other words the steady current deflection will have been reduced only 2.8 per cent by the vanishing of the middle sixth of the field. Now in an actual galvanometer the field strength in the region of the perforation is by no means zero, but may be as great as half the maximum.

If this deduction rests on a secure physical foundation it is evident that the height of the pole tips may be reduced much below the dimensions of Type B without material effect on the sensitivity. If, for example, the ratio of total height to diameter of perforation be made 3:1, the sensitivity will be reduced about 11 per cent, assuming zero field strength over the middle third. This points to the possibility of a material reduction in the size and weight of the galvanometer without much sacrifice of sensitivity.

2. *Consideration of Experimental Data.* As soon as this implication of theory was realized, all available experimental data which might serve as a check on it were carefully studied. The effective field of the Columbia University copy of the Leyden Model was carefully determined some ten years ago by the heavy wire method described in Section 2, §1. For comparison there was also available a series of determinations of the maximum field strength of this instrument measured with a ballistic galvanometer and exploring coil. The accompanying table gives a series of values by each method.

TABLE 1. *Comparison of maximum and effective field strength. Columbia University copy of Leyden Galvanometer*

Exciting current	Maximum Field, Ballistic Galv.	Effective Field, Heavy Wire
2 Amperes	17,100 gauss	16,800 gauss
3 "	19,700 "	19,400 "
4 "	20,800 "	20,700 "
6 "	22,000 "	21,900 "

Bearing in mind that the field is of considerable strength in the region of the perforation, whose length in this galvanometer is about one tenth of the height of the polepieces and that the possible error of determination of the effective field by the heavy wire method is one per cent while that of the ballistic galvanometer method as used in this work was nearly two per cent, the correspondence between the two sets of observations is such as to suggest that the theoretical conclusion is not far from the truth. One experimental observation seems to be in distinct contradiction. Einthoven (18), pp. 506–07, has calculated the strength of the effective field of his instrument from the difference between the motions of the middle point of a silvered quartz string on open and on closed circuit. He obtained the result that for a maximum field strength of 22,500 gauss, the corresponding effective field strength was 17,600 gauss. In working up the results of this set of observations it was assumed that the shape of the string when deflected by steady current is half of a sine curve.[19] It is also assumed that the difference in rate of decay of the motion of the middle point of the string on open and on closed circuits is proportional to the average velocity of all points of the string, or what amounts to the same thing,

[19] (18), p. 504, Einthoven says, "When the middle of the string moves with a velocity, v_1, one may put for the average velocity of all parts of the string $2/\pi v_1 = .637 v_1$. Now $2/\pi$ is the average value of a sine curve between the limits zero and π, so that it is evident a sine form is assumed.

that the electromagnetic damping at every point is proportional to the velocity as is the air damping. At the time this work was published Einthoven regarded his results in the light of a theory which assumes the motion of the string similar to that of a system with one degree of freedom. This is evident from his use of an ordinary differential equation to define the motion.[20] The assumption of a sine shape for the string follows naturally from this theory. Although the results of this simpler theory often lead to conclusions very little different from those reached by a more general analysis, it is not altogether safe to follow it when the quantities in question are small. Moreover, the equations which apply to the electromagnetic damping were not developed until eight years later.[21] The equations upon which Einthoven's calculation was based[22] were developed in connection with the theory of torsional instruments. These (unless the suspension is bifilar) are systems with one degree of freedom and the electromagnetic damping is strictly proportional to the angular velocity of the system. The theory of these instruments is so well known that it has been quite common to apply to the string galvanometer ideas which have become familiar in connection with torsional instruments. Crehore was similarly misled[23] in the treatment of electromagnetic damping. In the string galvanometer the velocity varies from point to point along the string, being greatest in the middle, while *the current generated by its motion is the same at every point of the string*. The electromagnetic damping is therefore identically the same at every point of the string. When the electromagnetic damping is appreciable the motion of even the fundamental mode becomes complicated, requiring for its expression a correspondingly less simple mathematical function (21, pp. 785–86). The electromagnetic damping of the partial of order s is $1/s$ of the magnitude of the electromagnetic damping of the corresponding amplitude of the fundamental (see Section 7). It might be possible by numerical substitution in Ornstein's equations to evaluate the true effective field from observations of the motion of the middle of the string, but without doing so it is evident that calculation of the results on the assumptions used by Dr. Einthoven must have led to a value for the effective field less than its actual value so that the result of this experiment cannot be regarded as necessarily in serious conflict with the theory of §1.

[20] (18) p. 494.

[21] (19), see reference to the work of L. S. Ornstein in Section 7.

[22] Taken from Kohlrausch, Lehrbuch der Praktischen Physik, 18, p. 486 and 494.

[23] 20, p. 213. See also Section 7.

3. *New Design, Type C.* Having become convinced of the probable soundness of the conclusions in §1, the writer was encouraged to design a new Einthoven galvanometer of very much smaller size than any hitherto described. By reduction in height of the pole tips it became possible to concentrate the windings without making the resistance of the field coils excessive. In this way the leakage is somewhat diminished and the bulk of it confined to the immediate neighborhood of the instrument, an obvious advantage. An experimental model of this galvanometer has been completed and its performance has been found to be in agreement with theoretical predetermination. Its weight is 5.8 kilos which may be compared with 18.15 kilos for Type B and 100 kilos for Type A. A further note describing this instrument will follow in the immediate future.[24]

SECTION 5. LIMIT OF SENSITIVITY AT WHICH THE DIFFERENTIAL EQUATIONS OF MOTION CEASE TO APPLY

It was pointed out in Part 1, p. 150, that the equations there developed will not apply after the tension becomes non-uniform. It is of interest to know at what order of sensitivity this limitation begins. To this an entirely unequivocal answer cannot be given without full development of a theory applying to the string when the tension is not uniform. For the present we may gain some practical insight into the matter by subjecting available experimental data to numerical computation. In the first place it is evident that if the string be slackened so far that it is not stretched beyond its "natural" length, the tension will be due to gravity only and at the upper end will be equal to mgl dynes. If the origin of Y be taken at the upper end of the (vertical) string, the tension at any point, P, whose ordinate is y, is given by $T=mg(l-|y|)$.[25] However much tension is put on the string, this non-uniform tension due to gravity is always present so that uniformity can be approximated only by making the impressed tension so large in comparison with the gravity effect that the latter becomes negligible in comparison. The numerical data which we are about to consider have been obtained from a galvanometer of Type B with a string 6.7 cm long. The linear density, m, for this string has been determined to be 2.42×10^{-7} gram/cm. (Appendix D.) In a particular experiment (Appendix C), the value of τ_0 was found to be 9×10^{-4} second. From

[24] This instrument was described and shown at the 137th meeting of the American Physical Society at Montreal, February 26, 1926.

[25] See E. J. Routh, Advanced Rigid Dynamics, 5th Ed., p. 379.

Part 1, Eq. (5), we have $T = 4l^2m/\tau_0^2$. From this and the numerical values given, we readily calculate $T = 53.7$ dynes as the tension at which this determination of τ_0 was made. This tension was sufficient to make the motion distinctly periodic, several highly damped oscillations taking place. The value of mgl, the limiting tension, we also readily find to be 1.59×10^{-3} dynes, from which we see that the tension in the periodic condition was some 34,000 times as great as the tension at the upper end when the string is fully slackened. It may be mentioned that the tension 53.7 dynes is about one tenth what was found to be a destructive tension for this string. This string was originally 7 cm long. In an attempt to make its motion as highly periodic as possible a tension of some 500 dynes was put on it, but before all the desired records were obtained it became nonconducting. It was resoldered and its length reduced 3 mm in so doing. It may be noted that 500 dynes would be a perfectly safe tension for this string so far as the tensile strength of the quartz is concerned, but the limit is determined by the tenacity with which the gold coating adheres to the quartz.

At critical damping the undamped period of this string was found to be 2.4×10^{-3} second (Appendix C). A calculation similar to the preceding gives for the tension when just aperiodic, 7.54 dynes. This is 4,700 times as great as mgl. The latter may surely be assumed negligible in comparison. Presumably a variation of one per cent in the tension would not vitiate the equations and we may inquire at what sensitivity this string would be under a tension equal to 100 mgl. We have (Part 1, p. 137) $\tau_0 = 2l\sqrt{m/T}$. Put in this equation $T = 100mgl$, using the values of m and l given above and taking g, the acceleration of gravity, as 980. We get for τ_0 the value 1.654×10^{-2} second. Now it is evident that if we alter the tension on a string and also alter the current so that, the field remaining constant, the deflection is kept constant, then the product of the current by the square of the natural period will remain constant (Appendix C). If we know this product for a given field and string at any tension, we may infer either term at any other tension provided we know the other at the new tension. In the experiment for which the value of the tension was calculated to be 53.7 dynes, $\tau_0^2 I = 8.6 \times 10^{-13}$. If we square the value of τ_0 obtained above for $T = 100$ mgl, and divide the number 8.6×10^{-13} by the result, we shall obtain the value in cgs units, of the current which will deflect the string under tension 100 mgl the same amount that the current I deflected it when under tension 53.7 dynes. The square of 1.654×10^{-2} is 2.736×10^{-4} and $(8.6 \times 10^{-13})/(2/736 \times 10^{-4}) = 3.14 \times 10^{-9}$ cgs units of current, or 1.04×10^{-8} ampere/cm. On the assumption that one per

cent variation in the tension is negligible, the equations still hold at this tension and the string is not yet like a vertical chain. Now it is of considerable interest to note that at just about this sensitivity a critical change in the behavior of the galvanometer is observed. At tensions slightly greater than 100 *mgl* the range of the linear deflection law is found to be smaller than it is at still greater tensions. This is no doubt due to the fact that the tension has now become so small that the increase with deflection is no longer negligibly small in comparison. A slight movement of the tension screw now affects the sensitivity markedly. As the sensitivity approaches that for which we have calculated the tension as of the order of 100 *mgl*, there is usually a slight shift of zero and focal plane, the sensitivity changes less rapidly for a given movement of the tension screw and the deflections again become proportional to the strength of current. The most reasonable explanation of these observations seems to be that the tension is now of the same order of magnitude as the internal stresses in the quartz core (metallic wires show the same phenomena, probably for the same reason), which is no longer constrained to remain in a straight line and assumes slight curvatures according to the magnitude and distribution of the stresses. Aside from stresses in the string itself, there may be introduced lateral and torsional stresses in the soldering of the ends of the string to the metal tips. This has been noticed by Einthoven (7). In making the strings one often discards quartz fibers which are distinctly curly and presumably even the best of those retained have something of this tendency. A string slackened until these stresses have been relieved is still under approximately uniform tension in something like the manner of a spiral spring of very coarse pitch. The change in length due to deflection may now occur by slight straightening of the stress curls without materially affecting the general uniformity of tension, hence the restoration of the linear law of deflection.

Since the equations show that the sensitivity varies inversely as the tension, we may infer that at a tension of 1000 *mgl* this string would have a sensitivity of about 10^{-7} ampere/cm. This may be of some interest as being about the sensitivity which is standard for the taking of electrocardiograms. With the particular galvanometer and string to which these figures refer, a sensitivity of 5×10^{-10} ampere/cm was easily reached. A tension of 10*mgl*, which would mean non-uniformity of tension to the extent of ten per cent, would correspond, if the usual equations still applied, to a sensitivity of 10^{-9} ampere/cm. From this it seems sufficiently evident that at the actual experimental limit of sensitivity the tension is distinctly non-uniform and that the behavior

of the galvanometer cannot be calculated with any certainty in the neighborhood of this limit by use of the theory for uniform tension.

In concluding this Section it may be mentioned that Wertheim-Salomonson has published a paper (21) in which he finds a mathematical expression for the sensitivity of the string galvanometer when the tension is relaxed as completely as possible. He treats the string in this condition by means of equations given by Lord Rayleigh for an elastic bar clamped at one end. It is open to question, however, whether it is admissable to regard the string in this way since it is fastened at the bottom and one cannot neglect rigidity at the bottom if the string is treated as a bar. He further assumes that since the sensitivity under uniform tension is proportional to the square of the natural period it is so also when it behaves as an elastic bar. This would seem to require proof. It is to be remarked, however, that the numerical results to which this theory leads agree fairly well in order of magnitude with the maximum sensitivities actually obtained with the best instruments hitherto constructed.

It seems to the writer that it would be, perhaps, more nearly in correspondence with actual conditions to treat the relaxed string by the equations for a rope or chain hanging under gravity.

SECTION 6. THE PERFORMANCE OF THE STRING GALVANOMETER

Much regarding performance may be inferred from the theory, but a prospective user of the Einthoven galvanometer will often wish specific information regarding results which have actually been obtained with available types of the instrument. In the papers cited Einthoven has given such data for the Leyden Model, and Type A described in this paper corresponds very closely in its properties and performance with his original instrument. The properties of any particular instrument will vary with the constants of the string which happens to be in use so that the values given should be regarded as representative only.

The length of the string in Type A is usually 13.5 cm and the resistance of these strings may be less than 1000 ohms and as great as 30,000 ohms. Usual resistances of strings for physiological and clinical use range from 3000 to 10,000 ohms. In the Columbia galvanometer a string 13.5 cm long and .0018 mm in diameter was found to be critically damped on short circuit when the current sensitivity was 2×10^{-7} ampere/cm with a magnification of 920 diameters. The resistance of this string was 8000 ohms. The value of τ_0 for this string at critical damping was .004 second. Substituting in Eq. (15a), Part 1,

τ_0 for t, we readily find that a steady current deflection will be nearly 99 per cent complete in a time equal to the natural period if the damping is just critical. If the instrument is to be used for measurement of periodic currents the natural period and steady current sensitivity in connection with the curve on p. 143, Part 1, will give a good index of the performance which may be expected. It is to be remembered in this connection that the response to a complex periodic current may differ slightly from what might be inferred from this curve since the curve represents the response of the fundamental mode only and the partial modes are somewhat underdamped. For critical work it would probably be well to prepare an experimental calibration curve by test with known alternating current. For most work it is best to use the instrument with the fundamental mode just critically damped. Strings whose mass differs from the one above referred to will give somewhat different results. For most work it is well to have them of very small diameter. The above data are for field strength of 20,000 gauss. With strings 13.5 cm long the period cannot be maintained at .001 second without danger of injury unless the diameter is very small. On the other hand it is advantageous to have the properties of the string such that critical damping will occur at about the tension which gives suitable sensitivity for the work in hand and this may sometimes require a heavier string. At a sensitivity of 10^{-8} ampere/cm the motion will be overdamped for almost any light quartz string in any good string galvanometer. As sensitivity is still further increased by slackening the string, the overdamping increases and the motion ultimately becomes very slow at the maximum sensitivity so that ten or fifteen seconds may be required to attain 99 per cent of the steady current deflection with the string as slack as possible. Theoretically the instrument could be much improved by having the string in an evacuated space at these high sensitivities. The experimental difficulties are considerable and I am not aware that anyone except Dr. Einthoven has had any extensive experience in the use of the galvanometer under this condition. As his work is not yet published I do not feel at liberty to discuss it. It may not be improper, however, to mention the fact that difficulties have been encountered in attempting to carry out this apparently simple procedure.

With the Leyden Model Einthoven obtained sensitivities of the order of 5×10^{-11} ampere/cm. At this sensitivity the stability of zero is remarkable. When using the galvanometer at such high sensitivities it is advisable to protect all parts of the circuit by electrostatic screening and it is often advisable to earth one end of the string. Mr. Louis G.

Pooler, of the Department of Physics, Columbia University, having occasion to work with a Type B galvanometer under circumstances which made earthing undesirable, found that he could prevent the building up of any considerable charge on the string by putting into the lower string cap a small tube containing the active deposit left by radium emanation. These tubes are easily obtained from any clinic where radium emanation is in use. Earthing is usually preferable, however. In case a string becomes charged it usually is attracted to and adheres to one of the microscope objectives unless it is under great tension. The procedure when this occurs is to increase the tension, then cautiously back away the lenses, watching the middle of the string through an opening usually provided for the purpose. Sometimes it is necessary to remove the string and replace it. This may injure the coating, but usually it does not. It is better to arrange the circuits so as to avoid these accidents. They are more likely to accur in winter when buildings are heated and dry. A metallic matting which may be earthed and on which the operator stands is often helpful. If a spark passes from the string to the microscope mounting the coating is usually ruined. Wiping the lenses is seldom necessary. If it has to be done the galvanometer should not be used with slack string for several hours. It might be supposed that deposition of a film of moisture on the lens by breathing on it would discharge it, but the writer has seen disturbance from this cause persist for several hours. The effect may be related to the phenomenon of residual charge well known in condensers. It must be remembered that the force required to deflect a slackened quartz fiber is extremely small. When greatly slackened the string is easily disturbed by air currents and before attempting measurements with it in this condition it is well to test the means provided for protection and supplement them if necessary.

Type B was not originally intended to be used with slack string, but no particular difficulty was experienced in reaching with one of these instruments a sensitivity in the neighborhood of 10^{-10} ampere/cm, the magnification being 1000 diameters in this experiment.

At critical damping its period will be about half that of Type A with a similar string, but at equal sensitivities the periods are of the same general order of magnitude. On account of the shorter string it can be operated at $\tau_0 = .001$ second with the average commercial string. With a magnification of 1000 diameters Type B instruments have a linear deflecton law for about 8 cm either side of zero. One such instrument in the writer's laboratory gives proportional deflections for 10 cm either side of zero when the string is slack. The large Edel-

mann instruments with strings supplied by the maker usually give proportionality for three cm either side of zero at 1000 diameters magnification. With a very good quartz fiber the writer has secured proportionality with this instrument for 4 cm, the sensitivity being 10^{-7} ampere/cm. Most commercial instruments have been handi-

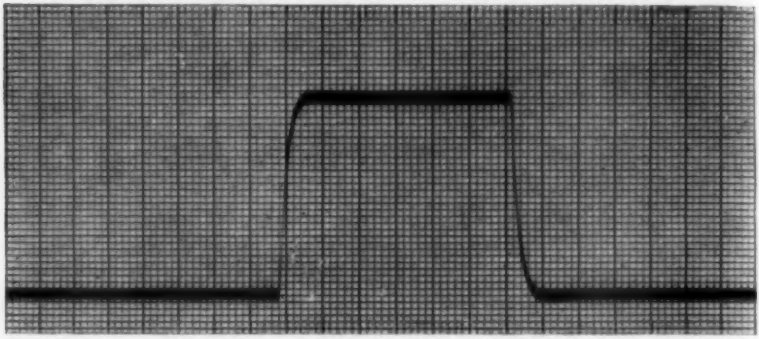

FIG. 7. a. *Damping barely less than critical. 6.3×10^{-1} ampere. The time co-ordinates are .001 second for one small division. $\tau_0=.0035$ approximately.*

capped by a very limited proportionality range. When the polepieces are not carefully made and the field is not uniform the deflections may become greater with increasing equal increments of current on one side of zero and rapidly less on the other side. This the writer has seen in an instrument whose polefaces were measurably inclined laterally. The

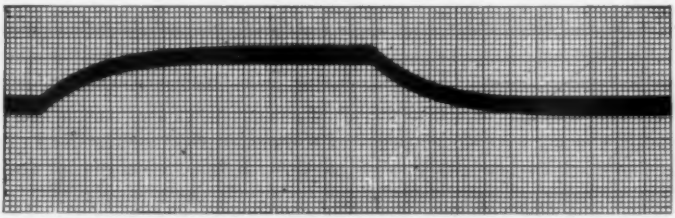

FIG. 7. b. *Damping very much greater than critical. 10^{-9} ampere per cm. Time co-ordinates .04 second for one small division. The magnification is 920 diameters in both. They were not made with the same string.*

weaker the field and the lower the magnification the smaller the linear range will be. For a given volt sensitivity it will be less the higher the resistance of the string, unless it be used quite slack, and less for a very heavy string than for a light one. Below are reproduced some photographic records which with their legends will serve to give further

information as to the average performance of these instruments. It will be realized that they are at their best in making measurements at moderate frequencies where considerable sensitivity is required. The writer constructed an instrument with string one cm long with which a natural period of .0001 second was easily attained, the sensitivity at that period being 10^{-4} ampere/cm, but air damping was not nearly sufficient to suppress the oscillations and the original plan to work with

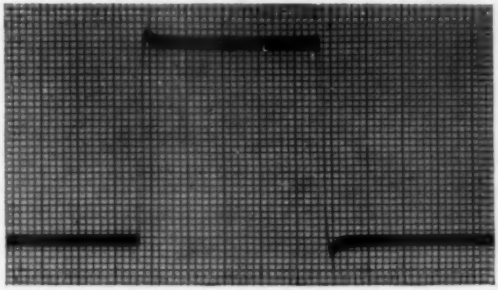

FIG. 7-c. *Showing the speed of deflection increased with moderate under-damping. Time, .001 second per small division.*

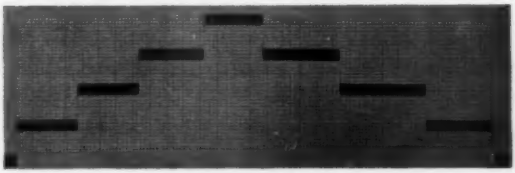

FIG. 7-d. *Damping critical. Shows proportionality of deflections for four equal increments of current and constancy of zero. Time, .04 second for one small division. The sensitivity is approximately 10^{-7} ampere/cm.*

low resistance strings so as to secure high electromagnetic damping was abandoned for reasons which will be discussed in the next section. By the use of liquid damping this instrument might be made serviceable, but on account of the addition of mass through dragging of the liquid, the sensitivity would be considerably reduced. There are several liquids of suitable viscosity whose refractive indices are such as would make possible the use of immersion lenses with corresponding increase of aperture. Thus by use of strings of small diameter and working at high magnification the loss of sensitivity might be partially made good.

W. F. Einthoven has described (14) a string galvanometer using strings a fraction of a millimeter long. The air damping is reduced to a minimum by putting the string in vacuum and the instrument is used tuned to resonance with the frequency of the current. It was used as a receptor of radio telegraphic signals and at the rapid rate of oscillation its movements would not be resolved on a rather slowly moving photographic paper so that the record appeared to consist of "disappearances" of the string, giving the effects of dots and dashes. It has been possible to attain natural frequencies as high as 300,000 cycles per second with this instrument.

SECTION 7. THE EFFECT OF THE CIRCUIT

In Part 1 the equations of motion were developed on the tacit assumption that the effect of the external circuit is negligible. Though this is never entirely true, it is so nearly true in the circumstances in which the galvanometer is usually employed that it was thought advisable to develop the theory in the form which seemed best adapted to direct attention to the more important features without undue complication. In the usual applications the resistance of the string is large and the external resistance is often large also. Under these circumstances the damping will be almost entirely due to the air. The inductance of the circuit is always small in the physiological applications and with a circuit of high resistance inductance would have to be large if its effect on the time of deflection is to become comparable with the mechanical lag of the string. If the string galvanometer comes in time to be applied to the measurements of physics and engineering, circumstances will likely arise in which the effect of the circuit can no longer be neglected. Since the war some of the sound ranging galvanometers mentioned in Section 3, §4, have been applied to some extent to current measuring problems. Trowbridge (22) has suggested that similar instruments may be used conveniently instead of the more usual oscillographs. It would seem fairly obvious to try to extend the range of such an instrument with vacuum tube amplifiers, and this has been done. In these instruments the electromagnetic damping is large compared with the air damping, exactly the reverse of the condition which obtains in the sensitive galvanometers with quartz strings. In the use of such instruments the effect of the external circuit cannot be neglected if the work in hand requires actual current measurement rather than the indication of time when a current begins as in the sound ranging application.

The first investigation of the circuit equations of the string galvanometer was made by Crehore (20). His work led Ornstein (19) to undertake a study of the subject. Ornstein's paper (in English) has apparently attracted little attention probably because it appeared when events at the beginning of the war were occupying attention and perhaps in part because of limited distribution of the Amsterdam Proceedings. This paper, which was unknown to the writer when Part 1 was written, deals with the motion of the string under electromagnetic damping. The treatment is original and rigorous and is undoubtedly the most compact and general exposition of string galvanometer theory available. It is, however, so much condensed that a reader of moderate mathematical ability will find it somewhat difficult to follow. In referring to Crehore's deduction of the circuit equation, Ornstein (p. 784) remarks, " . . . . not entirely exact energetic considerations underlie this deduction." This seems to refer to the following statement in Crehore's paper (l.c., p. 213), "Since the back electromotive force is proportional to the velocity, we may write, $e' = h_s' \varphi_s$, where h_s' is a constant quantity . . . . ." As mentioned in Section 4, §2, ideas derived from consideration of the moving coil galvanometer seem to have led many people to look upon the electromagnetic damping of the string galvanometer as simply proportional to the velocity. That this cannot be so was first pointed out to the writer in conversation by Professor H. W. Webb. Ornstein shows (l.c., p. 785) that for the partials the electromagnetic damping will be less the higher their order. This appears from his analysis, but is evident from physical considerations without mathematical development, a fact to which he draws attention. The adjacent segments of a string vibrating in parts move in opposite directions and generate emf's of opposite sign, so that of a train of odd partials *only one segment* contributes to electromagnetic damping. In Part 1, p. 160, reference is made to the motion of a metallic wire under electromagnetic damping. It was observed that when the "fundamental mode"[26] of this wire was more than critically damped, the third partial was still strongly in evidence. In Fig. 8 are represented two records of the motion of this wire. In one it is oscillating on open circuit and the decrement is small. In the second record the "fundamental" is somewhat overdamped. The third "partial" can be distinctly seen and in the original record the fifth can be distinguished also. At the time of writing Part 1 no satisfactory explanation of the

[26] Bearing in mind the results of Ornstein's paper it is perhaps inadmissible to speak of a fundamental mode in the usual sense. Something approximating it exists, however.

persistence of these partials could be advanced, but in the light of Ornstein's results exactly this would be expected.

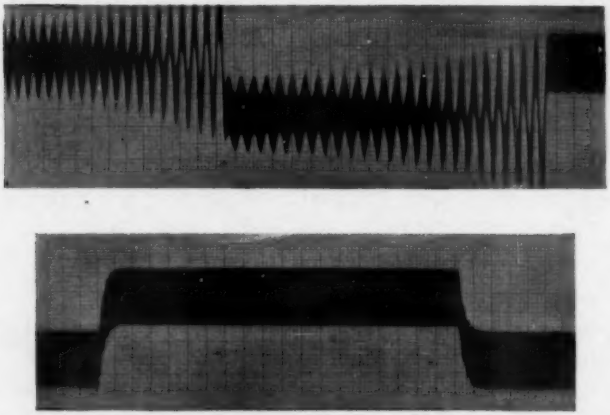

FIG. 8. *Motion of Duralumin wire.* a. *With air damping only.* b. *Heavily shunted. Time co-ordinates .001 second for one small division. Note effect of partials on the wave crests in* a. *Also the appearance of the third partial on the curves of growth and decay of the overdamped fundamental in* b.

Since with predominant electromagnetic damping the whole force available for damping the total motion is proportional to $\sum_{s=1}^{\infty} 1/s$ of all the emf's generated, it is obvious that the remarks as to resonance of the partials, Part 1, p. 163, will not apply in these circumstances. It will also be evident why the cruder galvanometers using heavy wires as strings cannot be made equivalent to galvanometers using light coated quartz fibers by the simple expedient of applying amplifiers. If, however, the string is damped with liquid and the external resistance so adjusted that the electromagnetic damping is small in comparison with the fluid damping, the heavy wire instrument may be made more serviceable for measurements of precision. If used with magnetic damping only, care must be exercised in interpretation of the curves.

It may be here remarked that the bifilar oscillograph, though usually treated theoretically as an instrument with one degree of freedom, in reality resembles very much a string galvanometer of low resistance with some added complication due to the coupling of the strings at the middle and loading them with the mirror. These instruments are usually used with fluid damping and at a natural period much shorter than that of the fundamental of the periodic current to be measured. It seems possible that in some applications the motion of the instrument may

be measurably affected by the occurrence in the impressed current of a component in agreement with one of the partial modes of the oscillograph system.

SECTION 8. ACCESSORIES TO THE STRING GALVANOMETER

1. *Illuminating Systems.* While the string galvanometer may be used subjectively and the motion of the shadow observed with an ocular provided with a scale, its peculiar field of usefulness like that of the oscillographs is in the investigation of phenomena whose variations are too rapid to follow with the eye. For photographic registration or for the use of a rotating mirror or similar device the instrument must be used as a projection microscope and some means must be provided for illumination. For many purposes a wire filament gas filled incandescent lamp is satisfactory. A simple condenser is necessary and it is advantageous to use a pair of lenses in order to diminish the spherical aberration. Small motor headlight lamps may be used, but they usually have a double filament. A single coil is preferable. It should be centered and then put out of focus until the field is uniformly illuminated with overlapping circles of diffusion. Definition of the string shadow will be poor unless the source of light is centered. Such a lamp may be used at normal voltage for recording the electrocardiogram, the magnification being about 500 diameters and the speed of photographic registration 25 mm/sec. By employing temporary overvoltage higher magnifications or higher speeds may be used. For very short exposures it is possible to secure quite satisfactory records on bromide paper at three meters a second. These lamps are inexpensive and have the advantages of requiring no attention after initial adjustment and consuming little current.

For still higher speeds up to ten meters a second the carbon arc is available. A smoothly operating automatic projection lamp using cored hard carbons is most convenient. If a really good automatic lamp is not available a hand feed lamp is preferable to a poor automatic. It may be operated on as small a current as 5 amperes, but the writer prefers to use larger carbons and more current, ten to fifteen amperes. The larger crater is steadier and there is less likelihood of its wandering enough to spoil a record. The best system of accessory lenses for use with an arc lamp is that described by Einthoven (7), p. 310. The condenser is a spectacle lens 3.5 cm in diameter and there is a second lens and two iris diaphragms. The entire arrangement is inexpensive and can easily be improvised in any laboratory. Einthoven arranges to illuminate the string with diffusion circles and so avoids sharp light

and dark bands across the records due to slight changes in the local temperature of the crater. This is desirable since, as will be explained presently, similar shadows are used to make the time record. A very great advantage of his system of illumination is that one of the iris diaphragms may be used to regulate the intensity of illumination, the other controlling the size of the illuminated field. These small lenses entirely fill the aperture of the microscopes. When Nernst lamps were available they were used extensively with string galvanometers. Tungsten arc lamps are in use to some extent. Aside from the initial expense and their rather short life they should be satisfactory.

2. *Photographic Registration.* For registration at slow speeds as in clinical electrocardiography, a variety of recording instruments have been developed by the makers of apparatus. Most of these have a magazine of film or photographic paper so that long continuous records may be made or they may be cut off at any desired point. The exposed paper usually accumulates in a box which may be removed and taken to a dark room for development. Others, as that devised by Mr. Lucien Bull, of the Institut Marey (made by Boulitte) carries the film or paper on a revolving drum similar to those used with oscillographs. The drum type is best for short records at high speed and for records of greater length at high speed one may use a drum which travels along a spiral. An excellent instrument of this type has been described by Eckhardt (23). The writer has used for many years a recorder described by Einthoven (24), adapted to all speeds from a millimeter a second to a meter a second. Apart from the fact that it is bulky this is a perfectly satisfactory instrument. It uses photographic plates. A variety of plate recorders have been devised in which the plate falls, uniformity of motion being attained in some by the principle of the Atwood machine and in most by damping. The damping is often obtained by using a pi ton displacing oil or air through a precision cock. Edelmann built a falling plate recorder in which the damping was obtained by rotation of a copper disk between electromagnets. An excellent discussion of the whole subject of photographic registration will be found in the article so entitled by the late Professor S. Garten in Tigerstedt's Handbuch der Physiologischen Methodik, (25). At the end of this article is a bibliography of 111 articles which probably includes everything of consequence in this field up to 1910.

Of the various available photographic materials paper is by far the cheapest and bromide paper can now be had which gives excellent results at magnifications of 500 diameters and speeds of two meters a second with a wire filament lamp. Slower papers are also available.

Reproduction of records is less convenient than with film or plates and neither film or paper is as convenient as a glass plate when precise measurements are to be made under a microscope. Plates can be had in every variety of speed and grain. They can be used in the falling plate type of recorder up to speeds of two meters a second and probably faster. Fast records of condiderable length require paper or film. Film may be had in long rolls either of the usual high speed emulsion, or with a slower emulsion such as is used for motion picture positives and called "positive film." This is excellent for sharp contrasts at slower speeds. Fast film will give good records at five meters/second with a magnification of 1000 diameters using a carbon arc lamp and legible records at ten meters a second.

Most inexperienced persons use too much light and too sensitive emulsions for photographic registration. The records should have a maximum of contrast and this is best secured by using the slowe t and finest grained emulsion and the smallest amount of light which will give results. The best optical definition is secured by using a source of high intrinsic brilliancy such as a carbon arc and cutting down the aperture with a diaphragm as in the illumination system of Einthoven. Since even the best objectives are imperfect in color correction it is well to ascertain by trial where to focus the subjective image sharply in order to get the best photographic definition. For a discussion of this matter and suggestions as to alignment of the optical systems reference may be made to a paper of Einthoven (26) on simultaneous records with several galvanometers.

During the war a detachment of engineer soldiers under the writer's direction constructed a number of photographic recorders of the magazine type with a device for automatic development and fixation. The records were developed and fixed in ten seconds and the device functioned with great reliability. Trowbridge has recently described a similar device for use at greater speeds (22). Military experience proved that photographic registration with suitable equipment is not only not difficult, but may offer the simplest means all things considered of solving a problem of graphic recording.

3. *Recording of Time Co-ordinates.* In the usual commercial application of the oscillograph it is customary to infer the time relations of the record from the average R.P.M. of the drum. If measurements of high precision are to be made this is unsatisfactory, not alone because the rate may change, but because the photographic material in the processes of development, washing and drying swells and shrinks. The most

recent information on the shrinkage of film is to be found in a paper in this Journal (27) by Curtis. In order to control with certainty the actual velocity of the record and at the same time to provide a reference system small enough to make the shrinkage negligible, Garten (28) devised the plan of photographing simultaneously with the record a system of rectangular coordinates. The space coordinates are generated by a series of lines ruled on the back of the cylindrical lens of the recorder. The time coordinates are shadows cast by the spokes of a wheel which rotates in such a position as to cut off the light at regular intervals. By making every fifth spoke thicker each fifth line is heavier. For controlling the speed of the wheel at a great variety of speeds Einthoven has described a very convenient device (29). For speeds not greater than one or two hundred mm/sec, the phonic wheel of LaCour (30) and Lord Rayleigh (31) serves very well. It was first applied for this purpose by Bull. The writer has used since 1912 a regulator described by Giebe (32) which was constructed at the Reichsanstalt, after a suggestion of Helmholtz, for controlling the speed of a rotary commutator used in measurement of capacity by the Maxwell bridge method. With two interchangeable regulator spindles this provides a large range of time intervals up to 1000 shadows a second. A modified form gave satisfactory results at 40 R.P.S. which with a 50-spoke wheel gives 2000 shadows a second. The co-ordinate network on the records published in this paper was recorded with this apparatus.

For the measurement of short time intervals by photographic methods with very high precision the tuning fork itself may be made to interrupt the beam of light in the manner described by Curtis and Duncan (35). For data on the accuracy of electrically driven tuning orks used in such measurements, see a paper by Dadourian (36).

APPENDIX A

ERRATA IN PART 1

At the beginning of this paper is a note calling attention to a defect in the statement of Eqs. (10) and (16) of Part 1, which was kindly brought to my notice by Professor H. W. March. In Eq. (10) the sign of summation should not have been used since its customary meaning would imply that all of the infinite number of solutions exist simultaneously, whereas they actually exist one at a time. The complementary function corresponding to any critically damped partial of order $s=lk/2\pi am$ is given by Eq. (10) with the sign of summation omitted. As a is altered by change of tension, $lk/2\pi am$ becomes equal

to each of the odd integers, s, successively. A similar remark applies to the second sign of summation in Eq. (11).

In Eq. (16) the limits of the sum should be from $s=1<lk/2\pi am$ to $s=(2n+1)<lk/2\pi am$. This equation will then apply to all modes of the string's motion for which s is less than the number corresponding to the order of the mode which is critically damped. For all orders of s above the one which is critically damped the motion is defined by Eq. (8). These corrections do not affect in any way the results of Part 1 since the interpretation put on these equations was that here stated as will be evident from remarks on page 159.

The following additional errata have been noted:

Page 144, fourth line above Eq. (14) read, *sth.* partial *is* critically damped. . . . .

Page 159, 4th line below Eq. (35) the statement that the fundamental decays at the same rate as the oscillations is not rigorously correct. The law of decay of the fundamental is given by Eq. (15a), p. 147.

Page 170, line 28, Eq. (III) should read, $u=e^{\beta t}\ (A+Bt)$.

Page 173, the last equation in Appendix h should read,

$$u=U\left\{e^{\beta_1 t}\left(\frac{\beta_2}{\beta_2-\beta_1}\right)-e^{\beta_2 t}\left(\frac{\beta_1}{\beta_2-\beta_1}\right)\right\}$$

Page 174, the date of Ref. (4) should be 1910.

APPENDIX B

DAMPED AND UNDAMPED PERIODS, DECREMENT AND DAMPING COEFFICIENT, k

1. *Period and Decrement.* When the relation between mass, damping and tension is such that the motion is oscillatory as defined by the complementary function of Eq. (8), Part 1, the damped period of the fundamental mode is easily determined. The string is deflected by steady current and if the effect of air damping alone is to be measured, a photographic record is made of the return to zero when the circuit is opened. At the same time there is registered a record of the time which may conveniently be in thousandths of a second. The period of one or several oscillations is then compared with the time scale under a micrometric microscope. From the damped period so determined we require the period the string would have if the damping were removed. The necessary formulas are readily developed from the results of Part 1. Although they are not new, the development, which is simple, will

serve to make clear their meaning. This seems desirable as confusion sometimes arises from the use of formulas based on the half period and others based on the whole period without explanation.

Let $u_1, u_2, u_3, \ldots$ be the amplitudes of successive whole oscillations of a damped train. If there were no partials the ratios $u_1/u_2, u_2/u_3 \ldots$ would all be equal. Einthoven found discrepancies which Crehore referred to persistence of the third partial during the first oscillation. Using a Type B instrument with a string of half the length of the Leyden Model it is possible to get much larger oscillations and in these circumstances the writer finds no discrepancy greater than the probable errors of measurement. The discrepancy observed by Crehore, whose curves were registered in the Columbia laboratory, was always that the first oscillation was smaller than it should have been had the motion been simply exponential. It is possible that some cause other than the persistence of the third partial contributed to increase the discrepancy in these records. At all events I have been unable to make certain of the effect of the partials in the smaller instrument though theory indicates it should be but little smaller than in the larger one.

If τ_k is the damped period, we have $u_1 e^{-k t/2m} = u_2$, from which it follows that

$$\log u_1/u_2 = k\tau_k/2m \tag{1}$$

The second member of (1) is called the logarithmic decrement and is usually represented by λ.

The analytical expression for the fundamental damped period (Part 1, p. 137), is

$$\tau_k = \frac{2\pi}{\sqrt{\frac{\pi^2 a^2}{l^2} - \frac{k^2}{4m^2}}} \tag{2}$$

Introducing this value of τ_k into the right hand member of (1), we have

$$\lambda = \frac{2\pi k}{2m\sqrt{\frac{\pi^2 a^2}{l^2} - \frac{k^2}{4m^2}}} = \frac{2\pi}{\sqrt{\frac{4\pi^2 a^2 m^2}{l^2 k^2} - 1}}$$

from which

$$\lambda^2/4\pi^2 = \frac{1}{\frac{4\pi^2 a^2 m^2}{l^2 k^2} - 1}$$

and from this by simple algebraical transformations we readily obtain

$$1+\frac{\lambda^2}{4\pi^2}=\frac{1}{1-\dfrac{l^2k^2}{4\pi^2a^2m^2}} \tag{3}$$

Multiply both terms of the right hand member of (2) by l/π, then

$$\tau_k=\frac{2l}{\sqrt{a^2-\dfrac{l^2k^2}{4\pi^2m^2}}}$$

which is identical with

$$\tau_k=2l/a\sqrt{\frac{1}{1-\dfrac{l^2k^2}{4\pi^2a^2m^2}}} \tag{4}$$

But $2l/a=\tau_0$ (Part 1, p. 137), whence from (3) and (4) we have,

$$\tau_k=\tau_0\sqrt{1+\lambda^2/4\pi^2}$$

For purposes of numercial computation the radical in (5) may be expanded by the binomial theorem, all terms after the second being negligibly small, and we have,

$$\tau_k=\tau_0(1+\lambda^2/8\pi^2+\cdots.) \tag{6}$$

The damping of a string galvanometer is always large so that under the maximum tension which will not injure the string there will be only a small number of oscillations before the amplitude falls off to a magnitude comparable with the unavoidable error of measurement. Instead of taking the ratios of the amplitudes of whole oscillations, we may determine the decrement from the ratio of any maximum to the absolute value of the next following minimum. The time of a half period is $\tau_k/2$. Writing for the logarithmic decrement of a half period λ', we have $\lambda'=k\tau_k/4m$, or $\lambda'=\lambda/2$, so that (5) becomes $\tau_k=\tau_0\sqrt{1+\lambda'^2/\pi^2}$ and expanding the radical,

$$\tau_k=\tau_0(1+\lambda'^2/2\pi^2+\cdots\cdot) \tag{7}$$

2. *To Determine the Value of the Damping Coefficient, k.*

1. When the motion of the string is periodic.

$$\lambda=k\tau_k/2m, \text{ whence } \quad k=2m\lambda/\tau_k \tag{8}$$

If the decrement is determined for a half period,

$$k=4m\lambda'/\tau_k \tag{9}$$

2. When the motion is aperiodic. We consider the fundamental mode only. The criterion of exact aperiodicity is $\pi a/l = k/2m$, whence $k = 2m\pi a/l$. But (Part 1, p. 137), $2l/a = \tau_0$, so that $2a/l = 4/\tau_0$, which gives,

$$k = 4m\pi/\tau_0 \tag{10}$$

APPENDIX C

CONSTANCY OF DAMPING AT HIGHER TENSIONS

Einthoven's experiment discussed in Section 3, §2, has shown that the damping, $k/2m$, is constant for a tenfold variation of tension when the absolute value of the tension is small. The following experiments indicate constancy of this ratio from a tension corresponding to critical damping to one approaching the maximum safe limit. The galvanometer used was Type B; length of string 6.7 cm, resistance 18,000 ohms. During the experiments the exciting field current was kept constant. The string was deflected by known steady direct current and the circuit then opened. The curve of return to zero was recorded on plates at one meter/second, with a time scale in thousandths of a second. At the greater tension there were several oscillations. The value of τ_k was determined by direct comparison with the registered time scale under a microscope provided with cross-hairs and a vernier stage. The amplitudes of successive maxima and minima of the oscillations were similarly measured. The ratio for the half period was found to be in one experiment 3.87. Log. $3.87 = \lambda' = 1.35$. $\lambda'^2 = 1.82$, $2\pi^2 = 19.7$, whence $1 + \lambda'^2/2\pi^2 = 1.0925$. The measured value of τ_k was .001 second. By Eq. (7), Appendix B, $\tau_0 = \tau_k/1.0925$. This will be the undamped natural period at maximum tension and to distinguish it from the undamped natural period at critical damping we shall add the subscript 1, so that $\tau_{01} = 9 \times 10^{-4}$ second. From Appendix B, (9) and (10) we have $k = 4m\lambda'/\tau_k$ for the periodic condition, and $k = 4m\pi/\tau_{02}$ for critical damping. Between these two equations k and m have presumably varied. We shall assume that over the range of tensions covered by the experiments the ratios k_1/m_1, k_2/m_2, are constant and see to what conclusion this assumption leads us. From (9) and (10) we then obtain $\tau_{02} = \pi\tau_{k1}/\lambda'$, where τ_{02} is undamped period at critical damping and τ_{k1} is damped period in the oscillatory state. Substituting the numerical values, we have $\tau_{02} = 3.14 \times 10^{-3}/1.35$ or, $\tau_{02} = 2.325 \times 10^{-3}$.

From Eq. (6), Part 1, we readily infer that with the same string in the same field, if the tension is varied and the deflection kept constant by simultaneous variation of the current through the string, the

product $\tau_0^2 I$ will be constant, τ_0 here referring to the prevailing tension, i.e., being regarded as a variable. We know the value of this product in the experiment under consideration for the periodic condition. It is $\tau_{01}^2 I_1 = 8.62 \times 10^{-12}$. If we divide this number by the square of the value of τ_{02} which we have calculated for the aperiodic condition, and if our assumption as to k and m is correct we shall obtain the value of the current which will produce the constant steady deflection U at the tension for which the string will be critically damped. We find $\tau_{02}^2 = 5.41 \times 10^{-6}$ and $(8.62 \times 10^{-12})/(5.41 \times 10^{-6}) = 1.59 \times 10^{-7}$ cgs units of current or 1.59×10^{-6} ampere. In registering the oscillatory curve the steady deflection of the shadow magnified 1000 diameters was three cm. Adjusting the tension until a three cm deflection was produced by 1.59×10^{-6} ampere a photographic record was again made and found to be aperiodic. A slight increase in tension caused barely perceptible overshooting of the equilibrium position so that exact aperiodicity had been very approximately attained. It is thus evident that within the limits of error of the measurements the assumption that the ratio k/m is constant holds good over this range of tensions also. Determinations of τ_{02} at critical damping were made from curves registered with this string on film run at three meters/second, using the method described in Part 1, pp. 147–48. The value so obtained was 2.4×10^{-3} second, in good agreement with the calculated value.

APPENDIX D

COMPUTATION OF VALUE OF m

By rearrangement of Eq. (6), Part 1, we have $m = HI\tau_0^2/32U$. We have shown in Section 3, §3, that τ_0 is determinate and unique in spite of variation of k and of the effective value of m. Knowing the true value of τ_0, we can readily calculate the true value of m, the linear density of the string. In a particular experiment the following values were measured,

$H = 23{,}500$ gauss.
$I = 1.22 \times 10^{-5}$ ampere or 1.22×10^{-6} cgs.
$\tau_0 = 9 \times 10^{-4}$ second.
$U = 3 \times 10^{-3}$ cm.

$$m = \frac{2.35 \times 10^4 \times 1.22 \times 10^{-6} \times 8.1 \times 10^{-7}}{3.2 \times 3 \times 10^{-2}} = 2.42 \times 10^{-7}$$

$m = 2.42 \times 10^{-7}$ grams per centimeter length.

APPENDIX E

MAGNETIC FIELD DATA

To present any considerable part of the data which has been accumulated during the progress of experimental work connected with the development of the new designs which have been described in this paper would probably not be worth the space required. It seems worth while to present some comparative data regarding the energy expenditure required to produce a given field strength in several different types and some figures regarding leakage. The effect of shortening the airgap in Type B and C is well shown by the three magnetization curves in Fig. 9.

TABLE 2. *Watts consumed for production of field strengths stated.*

Field Strength in Gauss	Type A	Type B	Leyden Model Columbia copy	Cambridge Old Type	Edelmann Large Model
8,000	1.5	0.37	1.5	7.0	7.0
10,000	2.5	0.63	2.5	12.0	12.0
12,000	3.5	0.96	4.0	18.0	20.0
14,000	5.0	1.35	6.0	38.0	25.0
16,000	6.0	1.81	8.0	40.0	70.0
18,000	8.0	2.16	13.0	65.0	160.0
18,000*	9.0	2.35	15.0	78.0	240.0
20,000	11.0	2.94	22.0	125.0	.....
21,000	15.	3.37	33.0	226	.....
22,000	19.	3.84	55.0	422	.....
25,000	90.	11.75	125+	.....	.....
26,000	250.	24.0	.....	.....	.....
27,000	.....	58.0	.....	.....	.....
28,000	.....	143.8	.....	.....	.....

* The Edelmann instrument has windings concentrated over poles 8.5 cm in diameter. This makes the average length of a turn rather long and the resistance considerable. The experiment stopped at the last value given because the accumulation of heat was so rapid that the insulation was beginning to char.

The above table illustrates the great advantage of suitable design. The distributed windings of the Leyden Model, Type A and Type B produce the necessary flux quite as certainly as the concentrated winding of the Edelmann Model. In fact the design of this instrument is such that no advantage can be taken of the concentrated winding since the accumulated heat endangers the insulation long before saturation is reached. In the old type Cambridge instrument there was more room for windings and the outer half was wound of heavier wire. The heat seems to have been better dissipated also as it handled 422 watts

without charring the insulation. Without water cooling it would be impossible to work with any such expenditure of energy and that it is quite unnecessary is shown by the production of the same field strength in the Leyden type at 55 watts. The production of a field in type B with an expenditure of 3.84 watts equal to that which required 422 in the older Cambridge type is partly due to the distribution of the windings, but largely to the shortness of the airgap which lessens the reluctance and diminishes the leakage.

Einthoven calculated the leakage factor for the Leyden Model to be 10.9 at a field strength of 10,000 gauss. That is, the stray flux will be 10.9 times the useful flux at that field value. The accuracy of his calculation is attested by a measurement which the writer made on the

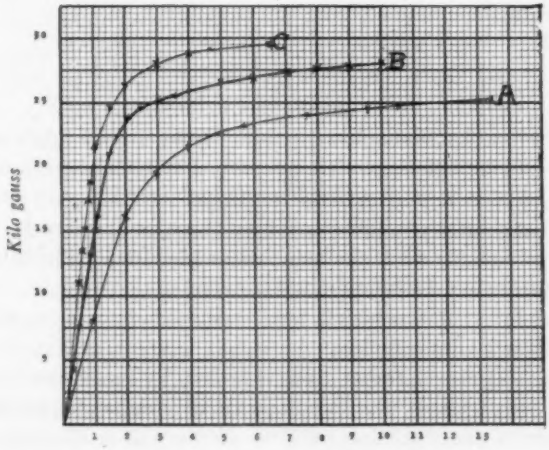

FIG. 9. *Magnetization curves for Types A, B, and C.*

Columbia copy at this field strength. The experimental value of the leakage factor was found to be 10.95. The leakage factor of Type B ranges from 5 at 10,000 gauss to about 8 at 27,000 gauss. The leakage factor in the old type Cambridge instrument must be very large, but I have never measured it.

Measurements of a Type B instrument show that the field strength in the region of the perforation averages about one third of the maximum value. In Type A and in the Leyden Model it is about one half the maximum value. The shorter airgap in Type B probably produces diminished leakage into the central region.

APPENDIX F

THE DAMPING COEFFICIENT

The damping coefficient, or coefficient of mechanical resistance, k, has been briefly considered in Section III, §4. In Section III, §2, the writer has referred to a theory developed by Stokes as probably explaining variations in the effective value of m which were observed by Einthoven. The theory of Stokes predicts that a cylinder of small diameter moving slowly in a viscous fluid will experience a large increase of mass and also of damping. Einthoven, after careful measurements concluded that k is constant over the entire working range of tensions. Regarding the precision of his measurements there is no question. Their interpretation rests, however, on the meaning of certain mathematical expressions derived from the equation

$$\ddot{u} + \frac{k}{m}\dot{u} + \frac{\mu}{m}u = 0 \tag{1}$$

Being an ordinary differential equation, this is strictly applicable only to a system with one degree of freedom, but for the purpose of the present discussion a more general treatment would merely complicate the expressions without adding anything of material interest. We shall not attempt to introduce the symbols employed by Einthoven (18) which are somewhat different from those used in these papers. The only new symbol in (1) is μ, which is here used to denote the coefficient of stiffness. When multiplied by the steady current deflection, U, the product, μU, is the restoring force which is in equilibrium with the impressed force during the continuance of steady deflection. The dimensions of μ are, therefore, $[MT^{-2}]$. In place of μ, Einthoven introduces $1/c$, where c is a coefficient of compliance or sensitivity.

Consider a photographic record of a steady current deflection, damping being either critical or more than critical. At the point of inflexion of the curve $\ddot{u}=0$. Introducing this value in (1), a solution for this point is at once obtained. This is essentially Einthoven's procedure. We have at the point of inflexion

$$u_i = -\frac{k}{\mu}\dot{u}_i \tag{2}$$

The negative sign may be disregarded as it refers only to direction of measurement. Of the quantities in (2), u_i is the ordinate of the curve at the point of inflexion and its value is read directly from the record.

The slope of the curve at the point of inflexion gives $\dot{u}_i$ and $1/\mu$ is the sensitivity of the galvanometer which is readily determined, so that k appears to be known. Although rigorously $\ddot{u}$ is zero only at a point, the velocity does not change rapidly in the immediate neighborhood of this point and it is conceivable that for a short stretch in this immediate neighborhood the motion might depend on an approximate equilibrium between the impressed force and the mechanical resistance. As stated, Einthoven finds k constant over the entire range of non-oscillatory motion by this method and, employing another method, he obtains almost the same value for oscillatory motion. He has reported one experiment, however, which can hardly be reconciled with this view of the matter. This experiment, already noticed in Section III, consisted in recording a deflection of the galvanometer at two sensitivities differing tenfold. The current was so adjusted as to give the same deflection in each case and for the lesser sensitivity the speed of the photographic plate was made ten times that at which the record of greater sensitivity was taken. The resulting curves are published in his 1903 paper (1), Figs. 1 and 3 of Plate III. They are of almost identical shape. The mathematical interpretation of this experiment is that the damping, $k/2m$, has remained constant. The range of sensitivities in these experiments covers the condition where Einthoven found large variations of m, so that k must have varied between these two conditions otherwise the ratio would not have been constant. The constancy of this ratio for small tensions of the string is decided by this experiment with a high order of precision and in Appendix C the writer has described an experiment which shows that k/m is still constant up to tensions which approach the limit of safe operation. While in general k and m for a cylinder moving in air would not vary in such a way as to preserve constancy of their ratio, nevertheless Stokes' theory indicates that for certain values of the argument this may occur.

As it appears impossible that k should be constant and equally impossible that Einthoven could have failed to detect the change which theory predicts will be large, the suspicion is aroused that perhaps Eq. (2), which appears to give the value of k, may for some reason not apparent on the surface, conceal the variations. Since the interpretation of the results of measurement depends entirely on Eq. (1), it is admissible to seek the answer to our question by analytical methods. As the interest centers about the point of inflexion, we shall commence by obtaining mathematical expressions for the ordinate of the curve at

this point, the time at which it occurs and the velocity at the point. We shall obtain these data first for the case of critical damping, for which the solution is (Part 1, Appendix f)

$$u = Ue^{-\Delta t}(1+\Delta t) \tag{3}$$

in which for brevity $\Delta = k/2m$. Differentiating (3), we obtain for the velocity,

$$\dot{u} = -U\Delta^2 te^{-\Delta t} \tag{4}$$

The negative sign in (4) occurs because (3) is written in the form which defines a decaying deflection. Differentiating a second time, we have for the acceleration,

$$\ddot{u} = -\Delta^2 e^{-\Delta t}(1-\Delta t) \tag{5}$$

The remark regarding the sign of (4) applies also to (5). The condition (other than $t = \infty$), for (5) to vanish is obviously $t = 1/\Delta$, which gives the time at which the point of inflexion will occur.

We must now obtain corresponding values for the overdamped condition. The solution of (1) for damping more than critical is (Part 1, Appendix h),

$$u = U\left\{e^{\beta_1 t}\left(\frac{\beta_2}{\beta_2-\beta_1}\right) - e^{\beta_2 t}\left(\frac{\beta_1}{\beta_2-\beta_1}\right)\right\} \tag{6}$$

where $\beta_1 = -\Delta + \sqrt{\Delta^2 - \mu/m}$ and $\beta_2 = -\Delta - \sqrt{\Delta^2 - \mu/m}$. Differentiating (6), the velocity is,

$$\dot{u} = U\left\{\beta_1 e^{\beta_1 t}\left(\frac{\beta_2}{\beta_2-\beta_1}\right) - \beta_2 e^{\beta_2 t}\left(\frac{\beta_1}{\beta_2-\beta_1}\right)\right\} \tag{7}$$

Differentiating (7) the acceleration is,

$$\ddot{u} = U\left\{\beta_1^2 e^{\beta_1 t}\left(\frac{\beta_2}{\beta_2-\beta_1}\right) - \beta_2^2 e^{\beta_2 t}\left(\frac{\beta_1}{\beta_2-\beta_1}\right)\right\} \tag{8}$$

The criterion (other than $t = \infty$) for (8) to vanish is,

$$\beta_1^2 e^{\beta_1 t}\left(\frac{\beta_2}{\beta_2-\beta_1}\right) - \beta_2^2 e^{\beta_2 t}\left(\frac{\beta_1}{\beta_2-\beta_1}\right) = 0.$$

Solving this for t, we readily obtain,

$$t = \frac{1}{\beta_2-\beta_1}\log\frac{\beta_1}{\beta_2} \tag{9}$$

Noticing that

$$\frac{\beta_2}{\beta_2-\beta_1}=1+\frac{\beta_1}{\beta_2-\beta_1},$$

(6) may readily be put into the form,

$$u=U\left\{e^{\beta_1 t}+\left(\frac{\beta_1}{\beta_2-\beta_1}\right)\left(e^{\beta_1 t}-e^{\beta_2 t}\right)\right\} \tag{6a}$$

Substituting the value of t from (9) in (6a), we obtain,

$$u_i=U\left\{e^{\log\frac{\beta_1}{\beta_2}\frac{\beta_1}{\beta_1-\beta_1}}+\left(\frac{\beta_1}{\beta_2-\beta_1}\right)\left[e^{\log\frac{\beta_1}{\beta_2}\frac{\beta_1}{\beta_1-\beta_1}}-e^{\log\frac{\beta_1}{\beta_2}\left(\frac{\beta_1}{\beta_1-\beta_1}+1\right)}\right]\right\} \quad \text{whence}$$

$$u_i=U\left\{\left(\frac{\beta_1}{\beta_2}\right)^{\frac{\beta_1}{\beta_1-\beta_1}}+\left(\frac{\beta_1}{\beta_2-\beta_1}\right)\left[\left(\frac{\beta_1}{\beta_2}\right)^{\frac{\beta_1}{\beta_1-\beta_1}}-\frac{\beta_1}{\beta_2}\cdot\left(\frac{\beta_1}{\beta_2}\right)^{\frac{\beta_1}{\beta_1-\beta_1}}\right]\right\}, \text{ or,}$$

$$u_i=U\left\{\left(\frac{\beta_1}{\beta_2}\right)^{\frac{\beta_1}{\beta_1-\beta_1}}+\left(\frac{\beta_1}{\beta_2-\beta_1}\right)\left(\frac{\beta_1}{\beta_2}\right)^{\frac{\beta_1}{\beta_1-\beta_1}}\left(1-\frac{\beta_1}{\beta_2}\right)\right\}, \text{ or,}$$

$$u_i=U\left\{\left(\frac{\beta_1}{\beta_2}\right)^{\frac{\beta_1}{\beta_1-\beta_1}}\left(1+\frac{\beta_1}{\beta_2}\right)\right\} \tag{10}$$

From (7), giving t its value in (9),

$$\dot{u}_i=U\left\{\left(\frac{\beta_1\beta_2}{\beta_2-\beta_1}\right)\left[e^{\log\frac{\beta_1}{\beta_2}\frac{\beta_1}{\beta_1-\beta_1}}-e^{\log\frac{\beta_1}{\beta_2}\left(\frac{\beta_1}{\beta_1-\beta_1}+1\right)}\right]\right\}$$

whence,

$$\dot{u}_i=U\left\{\left(\frac{\beta_1\beta_2}{\beta_2-\beta_1}\right)\left(\frac{\beta_1}{\beta_2}\right)^{\frac{\beta_1}{\beta_1-\beta_1}}\left(1-\frac{\beta_1}{\beta_2}\right)\right\}$$

or,

$$\dot{u}_i=U\beta_1\left(\frac{\beta_1}{\beta_2}\right)^{\frac{\beta_1}{\beta_1-\beta_1}} \tag{11}$$

Eq. (2) may be written as an explicit statement of the value of k,

$$k=\mu u_i/\dot{u}_i \tag{12}$$

Critical damping is defined by the relation $\Delta^2=\mu/m$, or $\mu=m\Delta^2$. Then from (3) and (4),

$$\mu u_i/\dot{u}_i=\frac{m\Delta^2 U e^{-\Delta t}(1+\Delta t)}{-U\Delta^2 t e^{-\Delta t}}=\frac{m(1+\Delta t)}{-t}$$

Whence, putting $t = 1/\Delta$ for the point of inflexion,

$$\mu u_i/\dot{u}_i = -2m\Delta = k$$

The significance of the negative sign as before is due to writing (3) in the form for a decaying deflection. We arrive at an equation which we might write, ignoring the sign, $k/2m = \Delta$, which at first sight suggests a mere identity. We shall show that Eq. (12) leads to the same result when applied to the overdamped condition. In this condition Δ still has the same constant value, but μ has diminished, say to μ_b.

From (10) and (11) we have,

$$\frac{\mu_b u_i}{\dot{u}_i} = \frac{\mu_b U\left(\frac{\beta_1}{\beta_2}\right)^{\frac{\beta_1}{\beta_1-\beta_1}}\left(1+\frac{\beta_1}{\beta_2}\right)}{U\beta_1\left(\frac{\beta_1}{\beta_2}\right)^{\frac{\beta_1}{\beta_1-\beta_1}}}$$

which immediately reduces to,

$$\frac{\mu_b u_i}{\dot{u}_i} = \frac{\mu_b(\beta_1+\beta_2)}{\beta_1\beta_2} \tag{13}$$

From the values of the β's we at once find $\beta_1+\beta_2 = -2\Delta$ and $\beta_1\beta_2 = \mu_b/m$, and substituting these values (13) becomes

$$\frac{\mu_b u_i}{\dot{u}_i} = -2m\Delta = k,$$

as was shown for the just aperiodic condition.

For the oscillatory case Einthoven obtained a value of k by determination of the damped period and the logarithmic decrement. (See Eqs. (8) and (9), Appendix B.) If we take the decrement for a whole period,

$$k = 2m\lambda/\tau_k \tag{14}$$

From Eq. (1), Appendix B, $\lambda = \Delta\tau_k$. Substituting this value in (14), we have $k = 2m\Delta$.

Although $k/2m = \Delta$ suggests an identity, it is not difficult to show that these equal ratios are by no means identical. Let Δ_a be the ratio for a string carrying a load of moving air and consider the case of critical damping. $\Delta_a^2 = \mu/m$. Should the m in this equation be also regarded as loaded with moving air or is it the true mass of a unit length of the string? The physical meaning of μ/m is perhaps best apprehended from the equation,

$$\tau_0 = \frac{2\pi}{\sqrt{\frac{\mu}{m}}} \tag{15}$$

In (15) τ_0 is the free period *without damping* and the mass which corresponds to the period without damping is obviously the true mass. So we might write the equation $k = 2m\Delta$,

$$\Delta = \Delta_a$$

where the first Δ corresponds to true mass and minimum value for the resistance coefficient. Similar considerations apply to the overdamped condition of which critical damping is merely a special case. Regarding the m in Eq. (14), it is to be noticed that Einthoven derived this from the free period without damping and the sensitivity data of steady current deflection with the string in equilibrium. This is also a value of true m. So Einthoven should have obtained exactly the result he did, constancy of the value of k for all tensions, since in all these equations the loaded values of k and m are never separated. The Eq. (1) was based on the assumption of constant values for these quantities and would not describe the motion of the string galvanometer even approximately were it not for the fact that Δ rather than its components is of importance. In investigating points on the curve of motion away from the point of inflexion, Einthoven puts the equation into a form such that k and m are separated. Here he at once experiences difficulty. Believing as a result of the measurements we have just discussed that k is constant he is forced to ascribe the entire discrepancy to variation of m.

We have shown that the finding of a constant value for k at all velocities of the string flows from the form of the equations by means of which the experimental results were treated. Properly interpreted these results do not appear inconsistent with the Stokes theory, on the contrary they support it. We may also remark that the result of the present investigation evidences the precision of Einthoven's experimental work.

It is perhaps to be regarded as fortunate that the ratio, Δ, rather than the coefficient, k, should have turned out to be constant since this renders it possible to use Δ in calculation and obtain correct results even though the components of Δ may vary in a manner not easily determinable.

APPENDIX G

ON THE PREPARATION OF FINE QUARTZ FIBERS

Aside from their use as the supporting core of galvanometer strings, fine quartz fibers are an important laboratory commodity. As many laboratory workers seem to be unfamiliar with a simple and convenient method by which they have been made for many years at the Columbia laboratory, it seems advisable to add a note on that subject. The bow and arrow method of making them described by Boys (33) is excellent for producing very long fibers, but requires much experience in preparation and handling of the equipment if fibers smaller than .003 mm are wanted. Nichols (34) described a method of blowing short fibers and Wertheim-Salomonson (8) described an elaboration of this method which permits of securing fibers long enough to serve as galvanometer strings. All of these methods have required a room flooded with sunshine to enable the operator to see and catch the fiber. Some ten years ago the construction of a new building adjacent to the writer's laboratory made sunshine unavailable except for about one hour daily and this led to the development of a method of working by artificial light which has proved more satisfactory than the use of natural sunlight. This method is a blowing process, but fibers more than a meter long can be produced and their fineness seems limited only by the patience and skill of the operator. Fibers .0015 mm in diameter are readily obtained by a reasonably good laboratory technician with a little practice. It is also possible to make much larger fibers. I have not tried to find out how large, but .01 mm presents no difficulty.

As raw material one may use the fused silica of commerce. Either the "silky" or transparent rods will produce good fibers. The type of oxyhydrogen lamp is important. Most of these will be found to be of the blow-through type with an inner tube for oxygen running out to the tip of the burner. While these can be used it is extremely difficult to adjust them and keep them in such adjustment that they will blow fibers with certainty. On the other hand a good mixing type of burner requires no adjustment and with very little practice gives uniform success. An excellent mixing burner was formerly supplied by Newton of London as part of a calcium light equipment and this I have found to serve admirably. Many of the older laboratories probably have these burners. Fig. 10 is a sketch of the illuminating apparatus. It is a wooden box one meter high, open at the top in the middle third. The middle part of the back and the bottom are covered with dead black fabric such as is used for light-proof window shades. The inside surfaces

of the front, sides, roof and lateral parts of the back are painted with white enamel. Six 75 watt nitrogen filled lamps arranged three on a side, furnish the light and give an intense, but diffuse illumination in

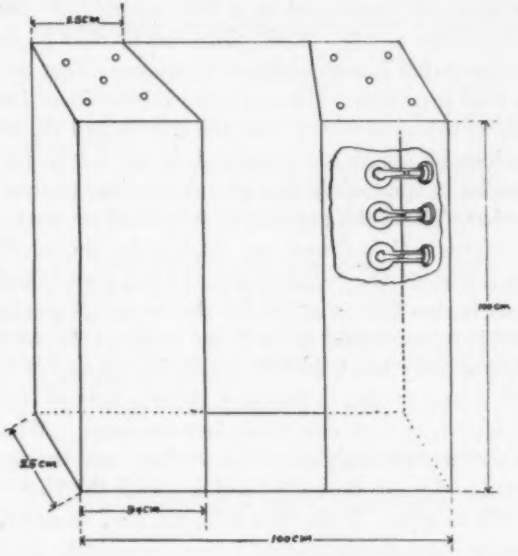

FIG. 10. *Sketch of illuminating box used in making quartz fibers.*

the middle third. At the back of the middle third is hung a piece of black velvet reaching the full height. There is a rather strong upward current of air in the middle of this box and in front of it due to heat

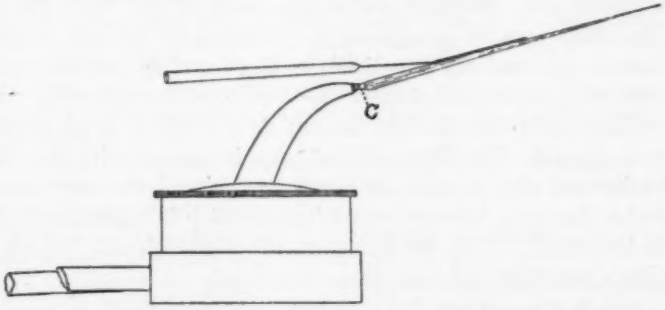

FIG. 11. *Method of blowing quartz fibers. "C" is the position of the "hot spot". The quartz softens, but does not fuse in the tip of the flame and is put under tension by the rush of gas. As the operator lowers his hand the rodlet melts when it reaches the hot spot.*

from the burner and the lamps and this air current serves a useful purpose as will presently appear.

The hot spot of the burner is at *C*, Fig. 11. A piece of rod is heated in the hot spot and drawn out to a thin, needle-like rodlet which is pulled apart at the middle. Each piece can be used to make a fiber. The size of the rodlet can be varied and to some extent determines the size of the fiber produced. The small end is introduced into the point of the flame as shown in Fig. 11 and the end toward the hand lowered until a considerable length of the rodlet is in the flame. Here again the length of rodlet in front of the hot spot determines to some extent the size of the fiber. The rodlet does not melt in the front part of the flame, though it softens and is drawn out straight by the moving hot gas. As soon as a part of the rodlet enters the hot spot it melts and the anterior part is shot like an arrow by the stream of gas across to the velvet where it will probably stick. If the hand is held steady, the fiber does not immediately break and the ascending current of hot air wafts it upward in a loop so that in this way fibers as long as two meters are sometimes drawn, though one meter is more usual. When it finally breaks the air current straightens out the loop and the operator may reach forward and grasp the rodlet and so secure the fiber. Sometimes the rodlet fails to stick. If the fiber is long it may be suspended in the air or blown up very slowly so that it can be caught. If short it will probably be blown upward so rapidly that it will escape. These events happen with a rapidity which is hardly conveyed by the description.

The flame should be smooth and almost noiseless and may be as long as ten cm. It is best to get oxygen in a cylinder at high pressure and use a reducing valve of the best quality. Illuminating gas is usually used with the oxygen. Heating quartz repeatedly in this flame causes a change which may be recognized by its losing its smooth ductility. As soon as this tendency is noticed, break off and discard the ends of the rods which have been heated. Do not allow the rods to lie where they will be contaminated while hot and do not handle the parts which are to be heated. The fibers will not come in contact with the velvet ordinarily and they should not touch any part of the apparatus as otherwise they may become contaminated which is important if they are to be coated. There are numerous other small details, but anyone working a few hours will learn them for himself.

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MARCH 20, 1926.

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